

Two-dimensional analysis of a drive sprocket mounting to a shaft using boundary element methods

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Abstract

This paper discusses application of the boundary element method (BEM) to preliminary stress analysis of a failed drive sprocket from a laboratory dynamometer. Initial inspections indicate the failure may have initiated in a slot key groove in the hub of the sprocket. The boundary element method, using BEASY, has been used to perform an initial, two-dimensional stress of the sprocket to evaluate the overall affect of the stress concentrations of the key slot. Additional work has been initiated to extend the work of this paper to three-dimensional models which will evaluate effects of other defects noted during a separate failure analysis of the sprocket. BEM model results will then be correlated with the final metallurgical-based failure analysis to recommend design, processing, and operating changes for the part.

Introduction

Shafts are mechanical devices used to transmit motion or torque from shaft to another. This is accomplished by means of gears, sprockets, friction wheels, pulleys, flanges, splines, or couplings. Turbomachinery, for example, will have a turbine shaft(s) driving a generator, generally accomplished by a direct coupling of the turbine shaft to the generator shaft through a bolted or splined connection. Or, a gas turbine or jet engine will have a shaft or several shafts driving compressors, fans, and accessories. Analysis of these shafts can be relatively

simple, using equations found in textbooks such as Juvinall [3] or Norton [4], to very complex analytical and test models which involve static and dynamic loading effects. Just as important as the design of the shaft itself is the design of the mounting of the gears, pulleys, sprockets, et cetera, to the shaft. Again, machine design textbooks such as Norton [4] contain description of design methodology for mounting components to shafts.

Failure of the shaft or component can be critical and expensive. For this reason, the design may contain a torque-transmitting device such as a key or pin, which, in many cases is designed to fail before the shaft or components fail; that is, the torque-transmitting device acts as a shear device, limiting the device to a “safe” torque. Furthermore, American National Standards Institute (ANSI) standards for design of keys are readily available to designers, and can be found in most machine design textbooks and handbooks. Figure 1 shows a typical key arrangement where the load is transmitted by horizontal forces. The fit of the key in the slot governs how the load is transmitted. In Figure 1-a, for example, a loosely fitted key, rotation of the key in the slot will result in pairs of diagonally opposite forces acting on the key. A tight fit will act as shown in Figure 2-b. Ultimate failure will occur as a shear failure shown by Figure 1-c. Forces are assumed to be uniformly distributed across the faces and equal to the shaft torque divided by the shaft radius. This is not entirely correct, but is a good engineering assumption for design.

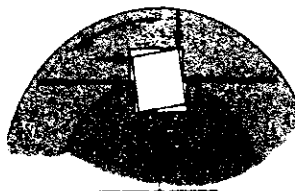


Figure 1-a

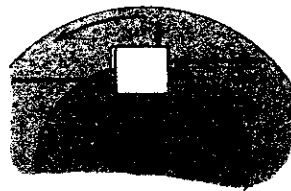


Figure 1-b

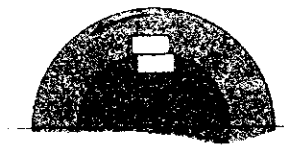


Figure 1-c

Stress analysis of keys is accomplished, again, through the use of “cookbook” type equations, obtained from texts such as Shigley [5], can be used to analyze the key. Referring to Figure 2 (a square key) for an applied torque(T), key of length(l), thickness (t), and shaft diameter (d), the resultant shearing force(F), shear (τ) and bearing (σ) stresses are:

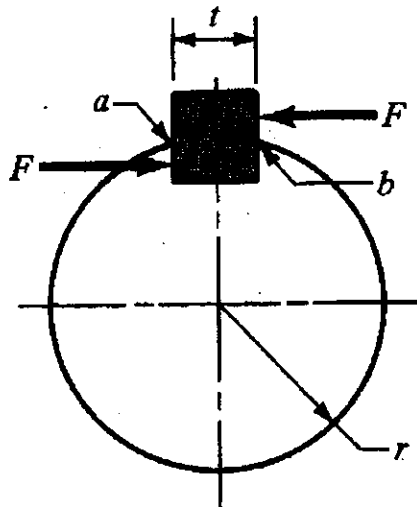


Figure 2 - Forces acting on Key

$$F = \frac{T}{d/2}$$

$$\tau = \frac{F}{tl}$$

$$\sigma = \frac{F}{tl/2}$$

These equations allow for evaluation of the key for shear and bearing (i.e., crushing of the key) stresses. Evaluation of the shaft or other components is accomplished by other means such as localized stress analysis of the shaft and sprocket or gear.

Of particular interest for this paper is a fractured sprocket, Figure 3, operating on a dynamometer, Figure 4. A failure analysis of the sprocket has been performed. Initial speculation, prior to a detailed failure analysis, was the stress concentration of the key slot was the source of the failure. Published analytical means of analyzing a sprocket for this type situation are not readily available. For example, a design manual for roller and silent chain

drives [2] contains limited data for design of a sprocket, none of which is really applicable to an analysis required for a failure of this type. Hence, a two-dimensional boundary element model of the sprocket was developed to study the effect of the key slot and its effect upon the design. Further studies, based upon a three-dimensional model are being developed.



Figure 3 - Failed Sprocket



Figure 4 - Assembly

BEM Model Development

Conditions which led to the failure are either unknown or very vague, complicating the scope of both the metallurgical and analytical aspects of the failure analysis. For example, the actual operating loads of the chain/sprocket drive are unknown. Using data from the chain drive design manual [2] and dimensions of the failed sprocket, it was assumed the design was rated for 100 pounds. Further complicating the procedure was the absence of the key. The size and material (strength) were therefore unknown to the investigators, adding another level of complexity to the analysis.

As previously mentioned, it was decided to initiate the evaluation using a two-dimensional model to gain insight into the effect of the key and its possible effect upon the failure of the device. With the absence of the key, and based upon some evidence the key may have been pressed (forced) into the slot, Figure 5, the BEASY model assumed a 5-mil interference fit on each side. The model is shown by Figure 6a. Solid modeling problems (generation of IGES files) negated modeling the teeth. A 100-pound load was therefore applied at the pitch diameter of the teeth. Later modeling will include the teeth. A square key was used with the edge held rigid as shown in Figure 6b.

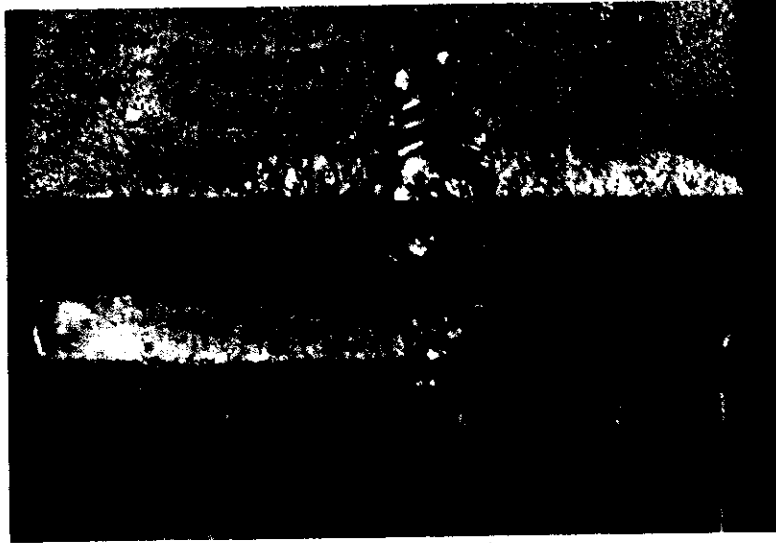


Figure 5 - Key Slot

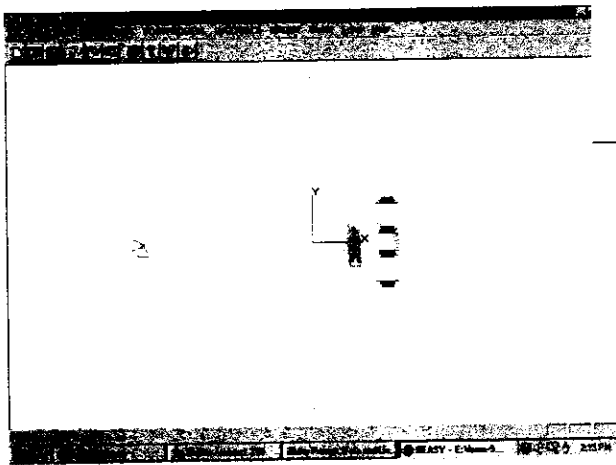


Figure 6a - 2D Model

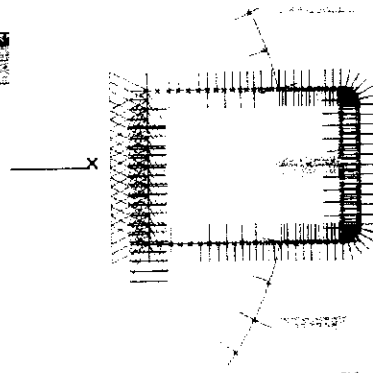


Figure 6b - Key Slot

The interference fit between the key and slot was modeled using gap elements (LINE_STRESS_BC, INITIAL_GAP).

Results

Results from the model are shown by Figures 7a, 7b, and 7c. Maximum stresses are in the range of 300-500 ksi, far exceeding the ultimate strength of the sprocket, assumed to be 80-ksi, based upon the hardness of the material. Hand calculations were performed assuming an interference fit of 0.0005 (one-half mil) on either side. When scaled, the analytical model and classical (hand) calculations are in agreement, with an average bearing stress of approximately 65 ksi. The hand calculations assumed both the key and sprocket had the same structural stiffness, which in reality is not true. Hence, the BEASY model correlates well with classical methods.

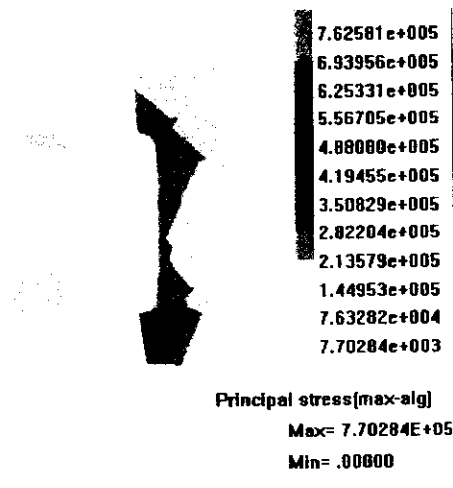


Figure 7a - Maximum Principal Stress

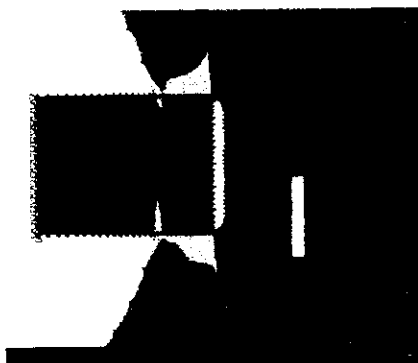


Figure 7b - Sig "X"



Figure 7c - Sig "Y"

Conclusions and Further Work

The two-dimensional model correlates well with classical calculations; and, with further development into a three-dimensional model provide accurate insight into the cause of failure. This in turn will provide information for corrective action to be taken in future designs, more likely, though, maintenance and operating practices to be used with the device.

Review of photographs of the sectioned sprocket, such as shown by Figures 3 and 5, indicate several potential problem areas. For example, there is evidence the retaining key may not have been fully inserted, effects of which are unknown and will be studied with the three-dimensional model. One sprocket tooth, not shown, has a deep gouge, the cause and effect of which is unknown, and will also be modeled. One tooth has a root crack, which, again will be evaluated. This three-dimensional model, including the set screw hole and the crack at the base of the hole, in the hub area, Figure 5, is currently being developed to obtain the "full picture" to be combined with the results of the failure analysis performed by others. A suspected overload condition can be evaluated by scaling the BEASY results, both 2D and 3D. In addition, loads can be placed at several different teeth to measure effects upon the failure area. The case of the partially inserted, and possibly oversized retaining key, can be evaluated to determine if the sprocket "cocked", causing the sprocket to wobble during operation, creating an unusual design condition. A final decision as to the cause of the failure and corrective action to be taken can then be made.

Acknowledgments

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