

Automatic 3D Crack Growth using BEASY

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Abstract

This paper described an automatic procedure for predicting crack growth in three dimensional structures as well as calculating the stress intensity factors at each crack increment.

The procedure is implemented in a general purpose engineering analysis system BEASY. The system uses a dual boundary element technique to simplify the modelling of the cracks and to provide highly accurate stress intensity factors. Applications are presented for single cracks and results compared with analytical solutions.

1.0 Introduction

In today's competitive marketplace, companies are seeking to improve the performance, quality and durability of their products in a short period of time. The complexity of modern products demands the use of **computerized** engineering analysis to optimize the product design and to increase competitiveness.

The use of computers for structural analysis reduces development costs by allowing prediction of stresses and displacements quickly and efficiently by varying design parameters such as geometry, loading and materials, without the need to build prototypes.

However, the prediction of stresses is only part of product design optimization. An additional critical requirement in the design process is to estimate the desired service life or engineering design life of the product. Analytically predicted fatigue life is therefore an important tool. A fundamental requirement for any durability assessment is a knowledge of the relationship between stress and strain and fatigue life for the material under consideration.

BEASY uses the Linear Elastic Fracture Mechanics life estimation technique to allow the designer or analyst to carry out fatigue calculations early in the design process. The fundamental assumption of linear elastic fracture mechanics is that the crack behaviour is determined solely by the values of the stress intensity factors which are a function of the applied load and the geometry of the cracked structure. The stress intensity factors thus play a fundamental role in linear elastic fracture mechanics applications.

In BEASY, crack-growth processes are simulated with an incremental crack-extension analysis.

For each increment of the crack extension, a stress analysis is carried out and the stress intensity factors are evaluated. The crack path, predicted on an incremental basis, is **computed by a criterion defined** in terms of the stress intensity factors.

The boundary element method is well established as a powerful solution tool for **fracture mechanics** (see Aliabadi & Rooke [1]). The reason for its success is the boundary only **representation**, the high accuracy and the ability of the method to represent the high stress fields near the crack front.

The theoretical foundation of the dual boundary elements and crack growth algorithm follows closely the work of Portela and Aliabadi [2] for 2D; and Mi and Aliabadi [3] for 3D. The authors will not repeat the theoretical basis but would refer the reader to references [2] to [5] for the detailed basis of the crack growth and dual boundary elements and to Brebbia and Dominguez [6] for a description of the boundary element method. The fatigue life predictions are evaluated using the generalized formula presented in NASA/FLAGRO 2.0 [7].

In this paper the implementation within the BEASY boundary element system is described and **simple 3D applications** with well know analytical solutions presented.

2.0 Simulation Strategy

A crack can be represented in a boundary element model using two main approaches. The traditional approach requires the user to define a zone interface surface along the crack surface. The problem is split into two zones and the edge crack is extended by a zone interface across to another external boundary. This technique while not requiring any special theoretical development places an extra burden on the user and introduces major complexity in the remeshing required for automatic crack growth.

The second approach uses the dual boundary elements to represent the crack. In this case the modeling is extremely simple and economical. The crack is represented by two surfaces occupying the same physical location, each surface representing a face of the crack.

2.1 Stress Intensity Factors

The system automatically computes the stress intensity factor using the crack opening displacement (COD) formula. This method can be used for computing Modes I, II and III intensity factors for edge and embedded cracks.

2.2 Incremental Crack-extension Analysis

The incremental crack-extension analysis assumes a piece-wise linear discretisation of the unknown crack path. For each increment of the crack extension, the dual boundary element method is applied to carry out a stress analysis of the cracked structure and the COD formula is the technique used for the evaluation of the stress intensity factors. The steps of this basic computational cycle, repeatedly executed for any number of crack-extension increments, are summarised as follows:

- Carry out a stress analysis of the structure
- Compute the stress intensity factors
- Compute the direction of the crack-extension increment
- Extend the crack one increment along the direction computed in the previous step
- Repeat all the above steps sequentially until a specified number of crack-extension increments is reached

For the sake of simplicity, the increment of crack extension is discretised with a fixed number of new boundary elements. In order to avoid numerical problems, concerned with the relative size of neighbouring elements, the crack increment length is kept between convenient limiting bounds, defined in terms of the size of the crack-front element. Apart from this constraint, the length of the crack extension increment may be defined by balancing the demands of accuracy and computational cost; the smaller the crack increment the more accurate and expensive is the analysis.

The results obtained from an incremental crack-extension analysis are the predicted crack path and the stress intensity factor variation along the crack path and along the crack front.

2.3 Crack-extension Criterion

Several criteria have been proposed to describe the mixed-mode crack growth. Among them, the most commonly used are the maximum principal stress and the minimum strain energy density.

The maximum principal stress criterion formulated by Erdogan & Sih [9] postulates that the growth of the crack will occur in a direction perpendicular to the maximum principal stress. Thus, the local crack-growth direction is determined by the condition that the local shear stress is zero. In practice this requirement gives a unique direction irrespective of the length of the crack extension increment.

The minimum strain energy density formulated by Sih [10] is based on the hypotheses:

- The direction of crack propagation at any point along the crack front is toward the region with the minimum value of strain energy density factor S as compared with other regions on the same spherical surface surrounding the point.
- Crack extension occurs when the strain energy density factor in the region $S=S_{min}$ reaches a critical value S_{cr} .
- The length r_0 of the initial crack extension is assumed to be proportional to S_{min} such that S_{min}/r_0 remains constant along the new crack front.

In 3D cases, the minimum strain energy density is adopted.

2.4 Crack Growth Relationship For Fatigue

A number of crack growth laws have been developed relating the rate of growth of the crack to the stress intensity factor. In order to satisfy a wide range of requirements, BEASY can use different methods to evaluate the growth rate.

One of the methods is the **generalized** NASGRO 2.0 equation [7]:

$$\frac{da}{dN} = \frac{C \cdot (1-f)^n \cdot \Delta K^n \cdot \left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{(1-R)^n \cdot \left(1 - \frac{\Delta K}{(1-R)K_c}\right)^q} \quad (1)$$

It should be noted that equation (1) may be reduced to the Paris equation [8] by setting the parameters p and q equal to zero and not considering the effect of crack closure, i.e. $f=R$ for $0 < R < 1$. The Paris equation is given by:

$$\frac{da}{dN} = C \cdot \Delta K^n \quad (2)$$

The database of fatigue material constants supplied by NASA and ESA has been integrated with the crack growth algorithm and this can be used to select standard data; or the user can create his own database containing the material constants.

Alternatively to the NASGRO equation, the user can define in an additional input file either a tabulated crack growth rate or the coefficients of the Paris, **Forman** or Rhodes equations. The tabulated data contains AK values for a series of da/dN and stress ratios R . For a given R and AK , the value of da/dN is calculated by using a linear interpolation of the two lines obtained from the log-log plot of $da/dN \times AK$. Each line corresponds to a certain value of tabulated R close to the input value of R .

3.0 Numerical Validation

The accurate prediction of crack growth is highly dependent upon the accuracy of the stress intensity factors. Comparisons of the stress intensity factors have been made with a number of published test solutions and the accuracy found to be very high. In the following examples, the stress intensity factors at each crack increment of the analysis are calculated and compared with analytical solutions. For real application, it is possible to **define** the material properties for fatigue crack growth analysis as well as applying irregular load spectra to represent complex loading histories.

3.1 Tube with an internal circumferential crack

The growth of a circumferential crack in the internal side of a tube under uniaxial load is investigated. The outer and inner radii of the tube are $R=2$, and $r=1$, respectively and the initial crack of length $a=0.2$. Figure 1 shows the initial model under a traction of 10MPa applied at the top of the tube and displacements fixed at the bottom. Due to symmetry only a quarter of the tube is modelled.

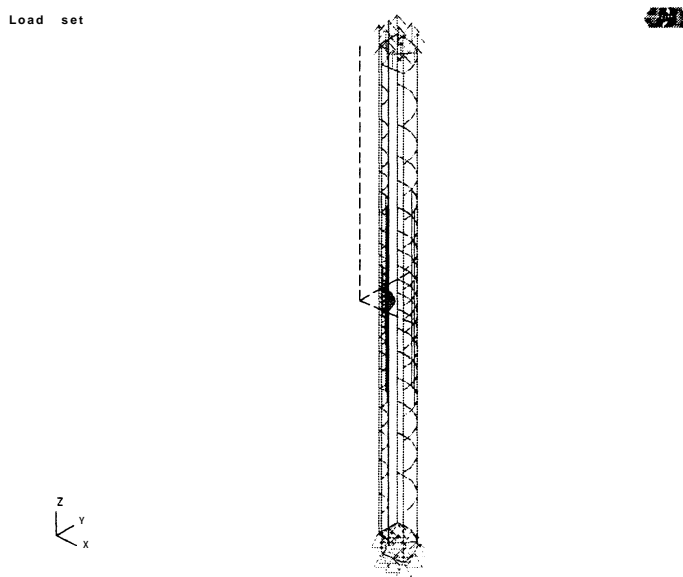


FIGURE 1. Complete model geometry of the tube with internal circumferential crack

A detailed view of the crack model is shown in Figure 2. The dual boundary element representing the crack can be clearly seen. The initial crack defined consisted of two elements in the radial direction. The additional elements have been automatically generated by the software as the crack grows. The stress intensity factor is computed along the crack front at each crack increment and the direction and growth increment predicted.

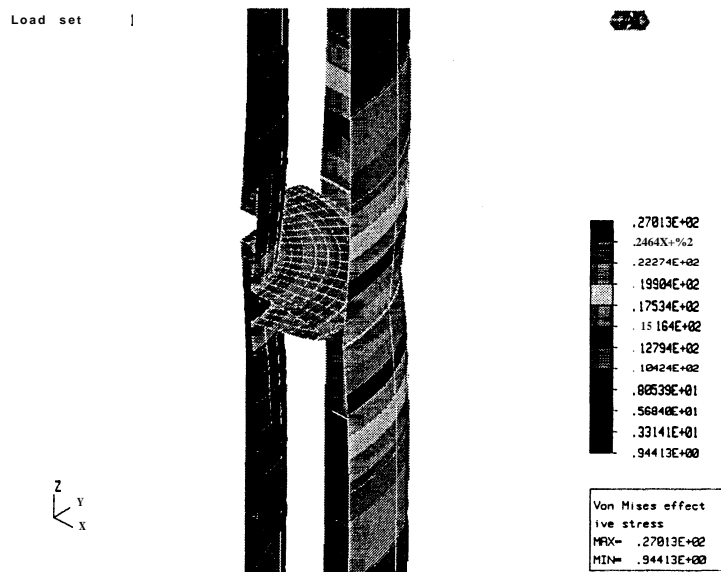


FIGURE 2. Von Mises Stresses near the crack

Table 1 compares the stress intensity factors for several crack lengths with the analytical solution given in ref. [11] which are based on ref. [12] and [13].

TABLE 0.1

Crack Length	KI (Beasy)	KI (Ref.11)
0.2	8.19	8.39
0.3	10.23	10.29
0.4	12.16	12.11
0.5	14.23	14.06
0.6	16.67	16.39
0.7	19.85	19.47
0.8	24.77	24.24

Figure 3 shows for each crack propagation the stress intensity factors along the crack front. As expected the values of K_{II} and K_{III} are equal to zero while the value of K_I is constant along the crack front and increases as the crack propagates.

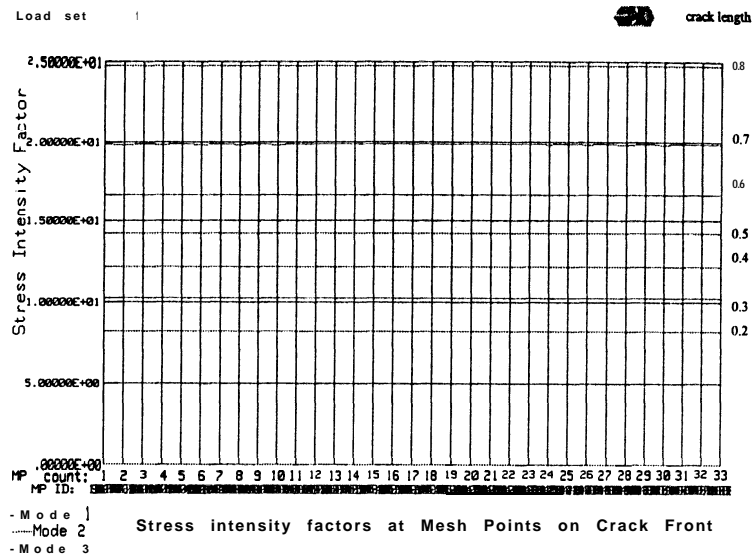


FIGURE 3. Stress Intensity Factor along the propagated crack front

3.2 Tube with an external circumferential crack

In this second example, the circumferential crack is on the external side of the tube. The outer and inner radii of the tube are $R=2$, and $r=1$, respectively and the initial crack is of length $a=0.2$. Figure 4 shows the initial model under a traction of 10MPa applied at the top of the tube and displacements fixed at the bottom. Due to symmetry only a quarter of the tube is modelled.

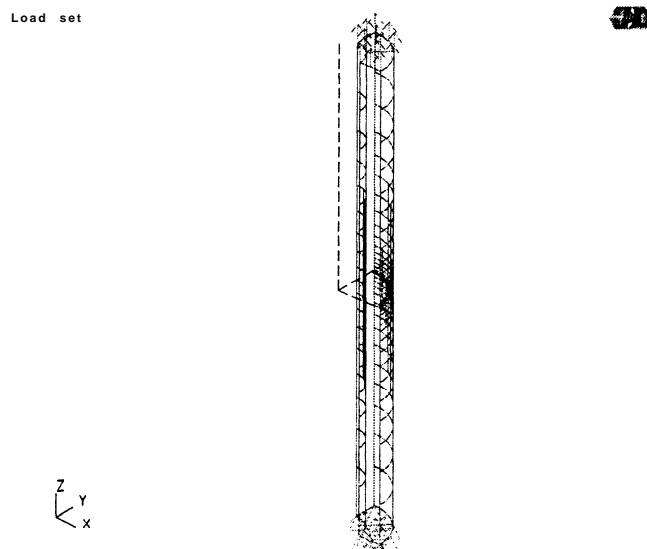


FIGURE 4. Complete model geometry of the tube with external circumferential crack.

A detail of the propagated crack can be seen in Figure 5.

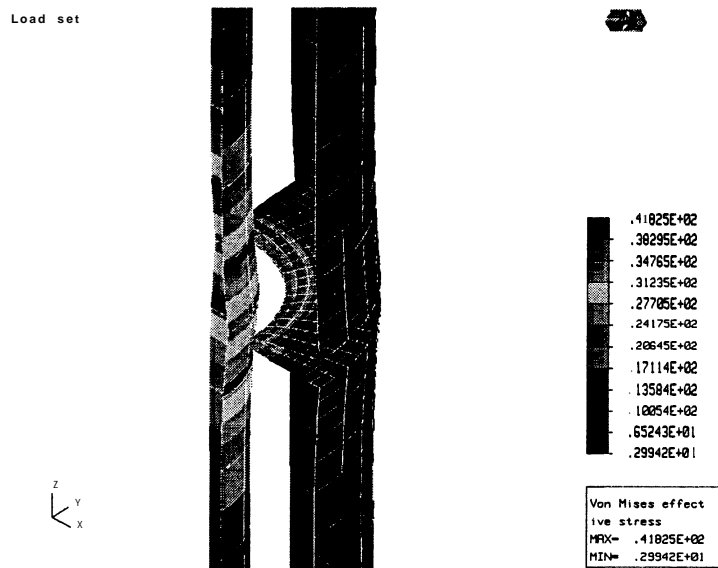


FIGURE 5. Von Mises Stresses near the crack.

Table 2 compares the stress intensity factors for several crack lengths with the analytical solution given in ref. [1 1] which are based on ref. [12] and [13].

TABLE 0.2

Crack Length	KI (Beasy)	K I (Ref.11)
0.2	9.52	9.72
0.3	12.64	12.72
0.4	15.93	15.86
0.5	19.71	19.49
0.6	24.38	24.07
0.7	30.73	30.42
0.8	40.67	40.54

3.3 Void with annular crack

A circumferential crack of initial length of $a=0.2$ around a spherical void of radius $R=1$. with far field uniaxial tensile stress $\sigma=10.0$ is considered. Figure 6 shows the Von Mises stresses for $a=0.4$.

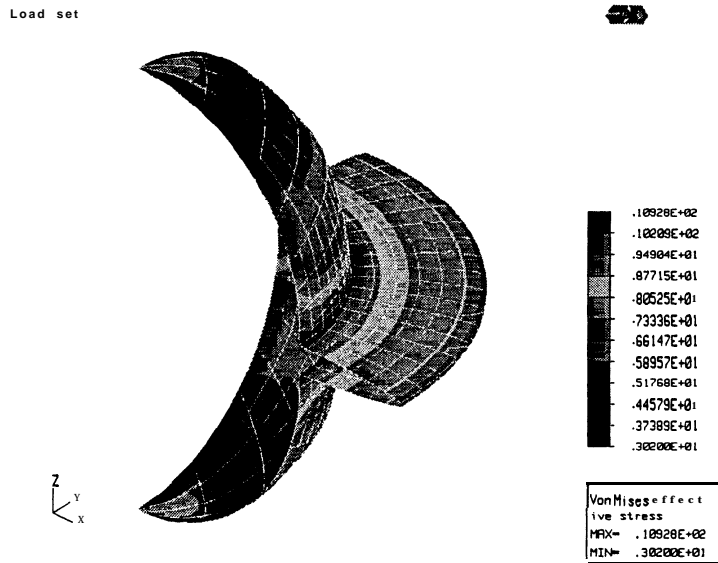


FIGURE 6. Von Mises stresses on a void with annular crack

Table 3 compares the stress intensity factors for several crack lengths with the analytical solution given in ref. [11] and ref. [14].

TABLE 0.3

Crack Length	KI (Beasy)	KI (Ref.11)	KI (Ref.14)
0.2	12.21	12.84	11.60
0.3	13.18	13.95	12.70
0.4	13.84	14.63	13.51
0.5	14.36	15.12	14.16
0.6	14.81	15.53	14.71
0.7	15.23	15.90	15.19
0.8	15.63	16.25	15.63

4.0 Conclusion

This paper has described a powerful system for the prediction of fracture data and fatigue crack growth in three dimensional structures. The techniques used provide high accuracy and the ability to automatically predict the growth of cracks. The system is particularly powerful because of the very simple modeling and complete lack of the meshing problems which are encountered if finite element method is used for crack modeling.

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