MODELING AND SIMULATION OF SPLINE COUPLINGS

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INTRODUCTION

In 1996 a project was established under the BGA Gear Research Foundation initiative with support from the DTI to develop new design and modelling technology for spline transmission joints. Improved shaft spline design methods were required to meet the needs of new civil aero engine designs that would have higher by-pass ratios and reduced core sizes. Such designs require smaller diameter shaft splined couplings that are capable of transmitting greater torque. Significant improvements in torque capacity may be required by future designs and will be realised by combined improvements in material strength and analysis techniques.

This paper describes the development of analysis techniques that can accurately predict surface stress levels for torsional and bending loading and provide an understanding of the elastic and load transfer mechanisms in spline couplings. Methods for generating spline geometry for BE analysis and determining the required tooth shape have also been developed. These methods will enable designers to carry out much more detailed analyses of their designs and permit optimisations to improve load capacity to be carried out.

The aim of the computational model was not only to predict the stress values on the surface of the spline and at other stress raising features but also to determine the distribution of the contact pressures on the spline and the magnitude of any stick or slip zones. The durability of the spline will be dependent not only upon fatigue but also the wear and fretting taking place on the contact surfaces. The distribution of contact stresses across a contact zone is dependent on the geometry and material properties of the components in contact. Although much experimental data is available on fretting and wear behaviour it is largely useless unless a detailed knowledge of the contact stresses and slip-stick behaviour is known.

The availability of predictions from the computational models for proposed transmission joint designs could therefore allow the exploitation of numerous fretting and wear experimental data.

THEORETICAL BACKGROUND

Major numerical tools like Finite Element (FEM) and Boundary Elements (BEM) have advanced to the extent that most engineering analysis problems can be solved. However contact type problems, where two or more objects come together to form a contact zone, have presented considerable difficulties. The main reason for this is that the contact area is not known in advance and in fact the position of the contact surface is part of the required solution.

Modelling spline couplings raises many technical issues. These have for example required development of a scheme which could accurate predict the contact stresses, slip distances, frictional effects, effect of tooth shape of the spline and the application of loading sequences consisting of torque and bending. This section describes the options and decisions made when developing the contact analysis capabilities.

A number of authors have published work on contact analysis using Finite Elements and Boundary Elements. The contact analysis is normally performed in three stages.

- An analysis which computes the displacement and stresses etc.
- A contact analyser that determines the contact conditions from the analysis.
- The implementation of the contact conditions.

In all methods the three stages are repeatedly applied until the contact solution is obtained.

Pascoe [2] has published a survey of techniques used by Finite Elements researchers and described some of the problems and limitations. In implementing a contact algorithm based on the Boundary Element Method (BEM) many of the lessons learned by FEM developers can be used as the contact problem is similar. In the contact area both methods have nodes and elements at which the contact behaviour is modelled. The main differences are that the boundary element model uses the surface traction as unknowns compared to the forces in the finite element method and that the BEM has elements only on the surface. For these reasons many of the common problems of FEM such as conversion of forces to surface traction's, working with stresses at the gauss points to avoid the negative stress problems at the corner nodes of three dimensional elements, overlapping displaced shapes on the surface are avoided.

In spite of the above many of the procedures required for boundary element contact analysis are similar to those used in finite elements. There are distinct advantages to the boundary element approach, which will be highlighted in the following

BOUNDARY ELEMENT CONTACT ALGORITHMS

A number of authors have published work on BEM contact analysis. The intention in this document is not to present a detailed description of the theoretical background of BEM contact analysis but to discuss some of the main issues and the results obtained using the BEM system.

Step 1 The initial analysis and load application

Assuming a problem has been defined which involves the contact of two or more bodies (or different parts of the same body) the first step is to perform a linear analysis. From the linear analysis the possible contact areas can be examined to check if any change in the configuration has occurred (e.g. areas come into contact or separated). If no change is necessary then the problem is effectively a linear contact problem and the problem has been solved.

There are three main approaches to starting the non-linear contact algorithm. These are:

- Applying the full load and then iterating until all the equilibrium and compatibility equation are satisfied. This approach is not satisfactory for most problems as it fails to follow the loading path and causes errors in the contact configuration.

- The second approach applies the loads incrementally where the size of the load step is determined by the size of load increment necessary to cause a change in the contact condition at any node. While this method accurately follows the development of the contact surface and eliminates the iterations it can cause excessive CPU times for problems with a number of nodes in the contact areas. This is caused by the very small load increments required. For problems with a small number of nodes in the contact area this approach is very successful.

- The third approach which was chosen for use within BEASY requires the load to be split into a number of load increments. The increments are predefined and adjusted automatically depending upon the rate of convergence of the solution. For each load step the solution is computed and iterations are performed until the compatibility and equilibrium conditions are satisfied.

Step 2 The Contact Analyser

Once a load increment has been defined and the analysis performed the displacement, traction and stress solutions are obtained at the nodes. The function of the contact analyser is to determine if the surface is under the following conditions.

- separated i.e. not in contact
- sticking contact
- frictional sliding contact
- frictionless sliding contact

If for example the current configuration is that the surfaces are in contact but the normal traction on the surface shows a tension stress then the function of the contact analyser is to generate a constraint which applies the separated condition. Alternatively if the tangential stress is greater than the frictional resistance then the frictional sliding contact condition is applied. Similarly the tractions and displacements can be used to determine the correct contact condition for any current values of the loads.

The BEM contact analyser benefits from the directly computed tractions and displacements on the contact surfaces, thus allowing accurate and precise determination of the contact zone and the contact forces necessary for friction calculations. No problems have been observed with interpolation problems with BEM, which is in contrast with FEM. The accurate tractions and displacements available in BEM are particularly beneficial on curved surfaces.

Step 3 Implementation of the Contact Condition

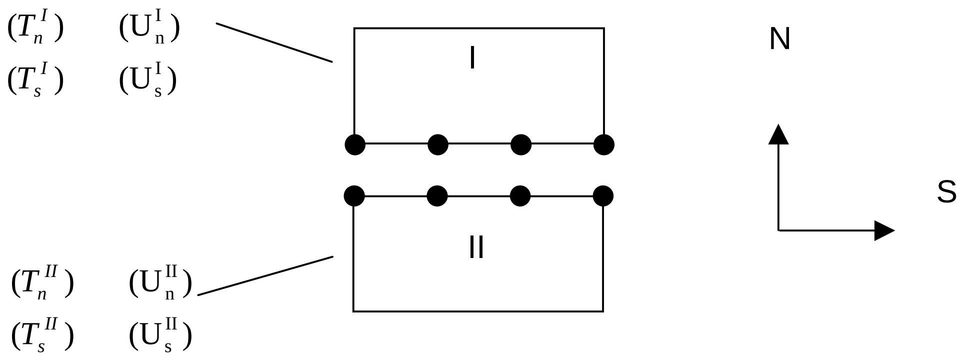
Once a new contact configuration has been identified it has to be tested to determine if the compatibility and equilibrium conditions are satisfied. This requires a solution of the analysis problem under the new contact boundary conditions (constraints). There are a number of approaches that can be used:

CONSTRAINTS BASED CONTACT ALGORITHM

The direct application of the constraints to the system matrix has a number of advantages. The extra equation can be added to the system matrix in such a way that only a partial reduction of the matrix is necessary for each iteration. This provides a major reduction in the computer time necessary so enabling more complex problems to be solved. The constraint method is also not restricted to node on node contact, and large amounts of sliding can take place. The contact analyser generates the constraints applied directly to the system matrix and they simply express the equilibrium and compatibility conditions on the contact surface.

Contact Implementation using constraints

Consider the following two, two-dimensional bodies in contact



The unknowns defined at the nodes on the contact surface are both the displacement (U) and traction (T) components on the surface. For example the normal (n) and the tangential (s) components of the traction T. The two components surfaces in contact are surface (I) and surface (II).

There are a limited number of configurations for the contact. E.g. Separated, In contact, stick, slip etc. The contact configurations can be defined exactly using the constraint equations. For example the constraint equations for the contact stick condition are:

The nodes are fully coupled

Compatibility

$$U_n^I + U_n^{II} = 0 \qquad (1)$$
$$U_s^I + U_s^{II} = 0$$

Equilibrium

$$T_n^I + T_n^{II} = 0 \qquad (2)$$
$$T_s^I + T_s^{II} = 0$$

Note this also applies when stick with friction when $T_s \leq U_s / T_n /$

SPLINE MODEL GENERATOR

In order to simplify the generation of the spline model a tool was developed to automatically generate the model using the typical design data.

- Number of teeth
- Tooth length
- Pitch
- Stub Pitch
- Pressure Angle
- Helix Angle
- Root and Cutter Information

Based on this data the software automatically creates a model of the male and female spline suitable for analysis.

The analysis model can either consist of a single tooth where cyclic symmetry is assumed, or the single tooth can be replicated to form a complete spline joint.

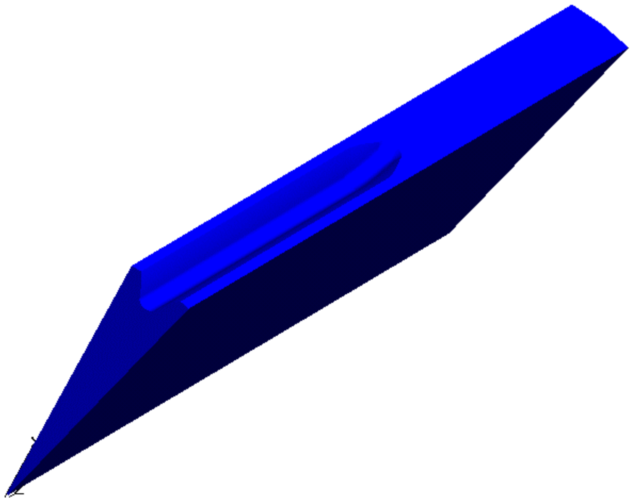


Fig 1 Cyclic Symmetric single tooth model of the male spline automatically generated using the spline generation tool

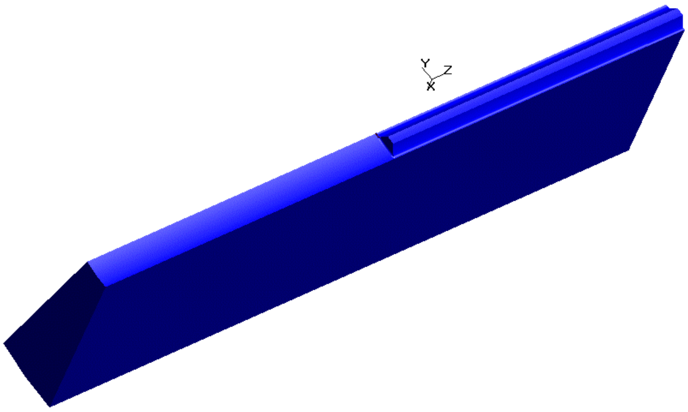


Fig 2 Automatically generated female single tooth spline

The single tooth model with cyclic symmetry can be used to predict the performance of the spline coupling under torque loading. However if the joint is subjected to bending loads or the model is to include manufacturing tolerances then a complete model has to be used. Rotating the single tooth model 360 degrees and replicating the teeth at each step can easily create this type of model. This can be performed automatically.

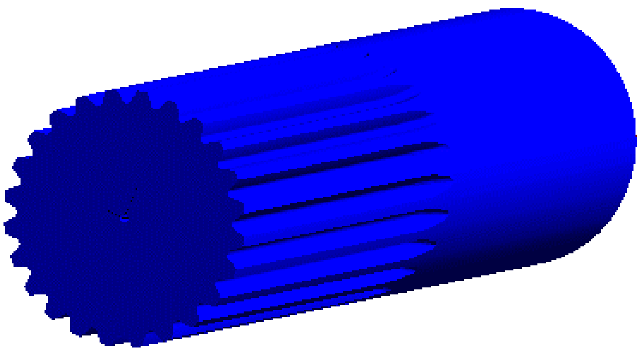


Fig 3 Complete male spline model generated by the spline generated based on the design information provided by the user.

Modifying the spline shape to reduce the contact stresses and other manufacturing processes can be imposed on the model by specifying the initial gaps etc between the contact surfaces.

APPLICATIONS

PUNCH MODEL

To validate the contact and friction algorithms, 2D and 3D models were created of a “benchmark” problem first studied by Ohte [1].

The mesh used for the 3D BEASY model of the benchmark problem is shown in figure 4. Symmetry conditions are applied at the plane x = 0, so no boundary elements are required there. A vertical load is applied to the top of the punch and the base of the foundation is restrained against vertical movement.

Figures 5 and 6 compare the distribution of normal pressure, relative tangential shear stress and tangential slip, for the BE 2D and 3D models and 2D FE results. There is close agreement between the BE and FE solutions. The change of slope at “distance along interface” about 0.5 in figure 6 reflects the change of regime from “stick” to “slip”, and confirms the friction algorithms are functioning correctly.

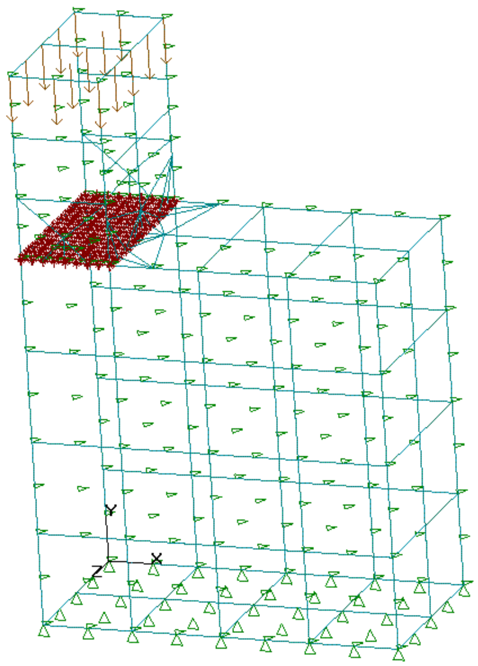


Fig 4 Showing 3D BEASY model of benchmark problem

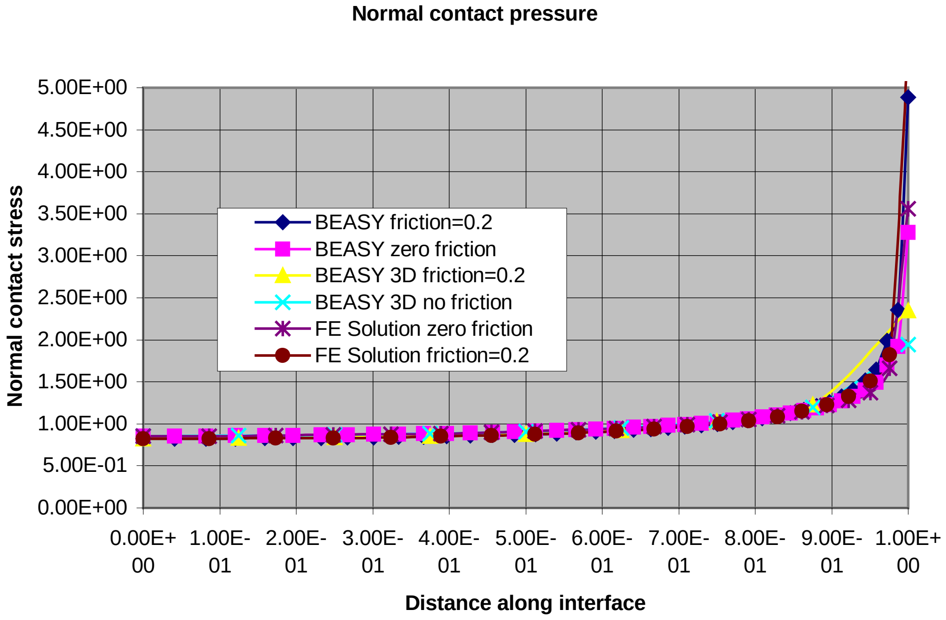


Fig 5 Comparison of normal contact pressure for friction coefficient =0.2 and zero friction, for 2D and 3D BE models and with 2D FE.

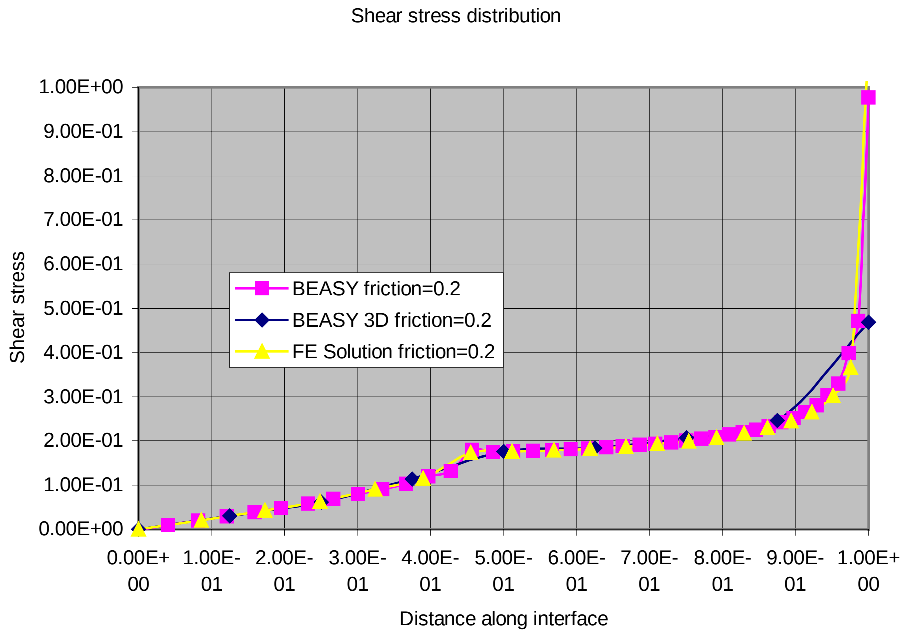


Fig 6 Comparison of shear stress distribution along the interface with friction coefficient =0.2, for 2D and 3D BEM models, and 2D FE

Helical 18 tooth spline joint

This example is of a helical 18 tooth spline joint. The spline tooth shape is such that in the unloaded state, there is contact over a short region towards the middle axial position of the teeth.

Cyclic Symmetry Spline Model:

A single tooth is modelled for both the male and female spline. Elements are placed on the cyclic symmetry surfaces, and cyclic symmetry is enforced. Loading is by tractions applied to the end of the male spline in the transverse direction, resulting in a torque load. The end of the male spline is restrained against axial movement. The end of the female spline is fully restrained.

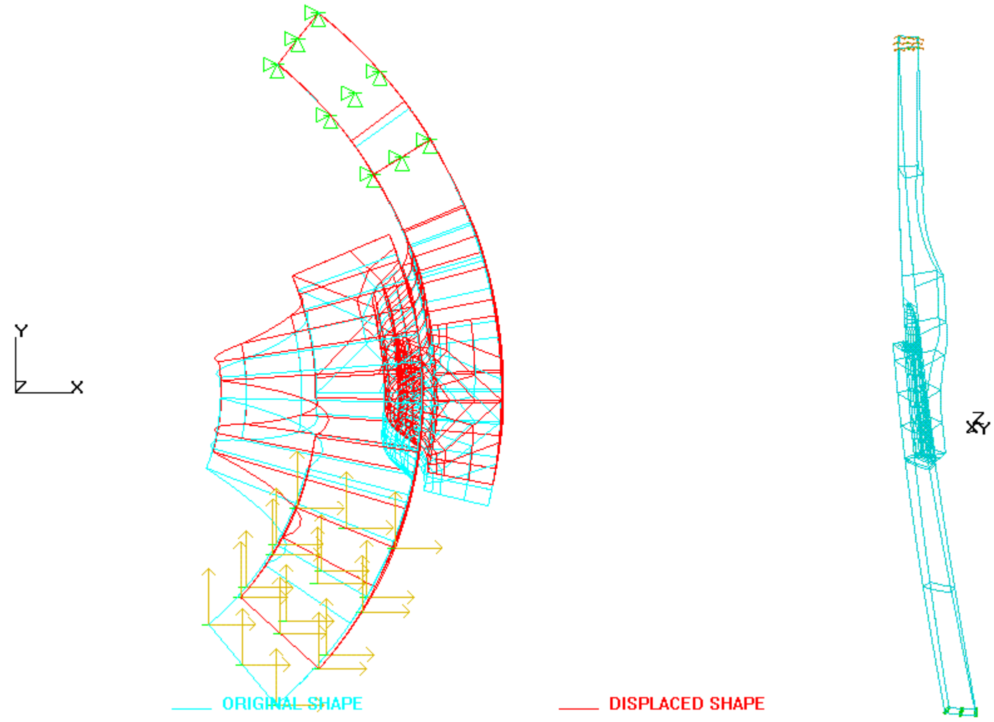


Fig 7 Cyclic symmetry mesh

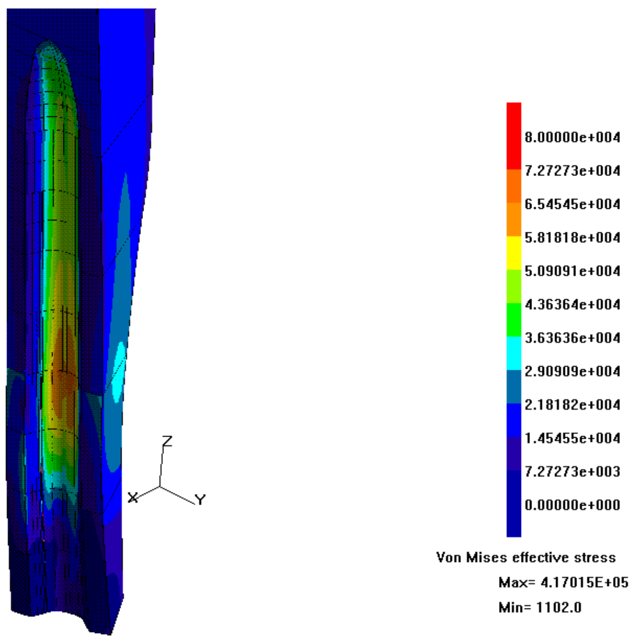


Fig 8 Cyclic symmetry model: Contours of Von Mises stress on deformed shape

Figure 7 shows the mesh of 398 elements and loading. Figures 7 and 8 show the displaced shape and Von Mises stress contours respectively.

Full Spline Model:

In this model, the complete male and female splines are modelled. Loading is by tractions applied to the end of the female spline in the circumferential direction, resulting in a torque load. The end of the female spline is restrained against axial movement. The end of the male spline is fully restrained.

Figure 9 shows the mesh of 3786 elements. Figures 10 shows the surface contact pressures.

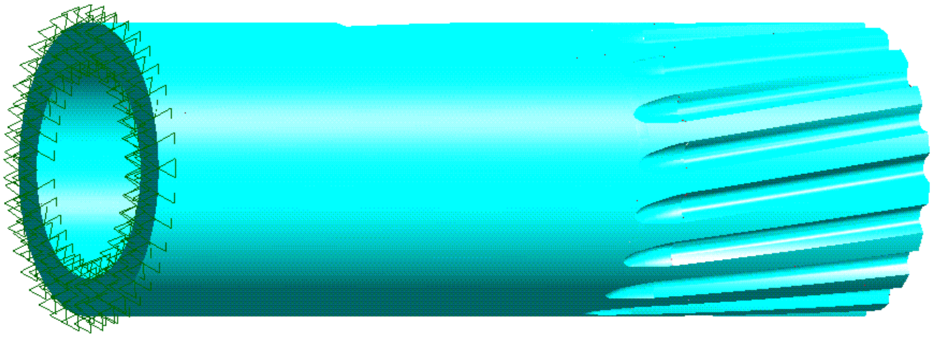


Fig 9 Male part of 18-tooth model. 3786 elements

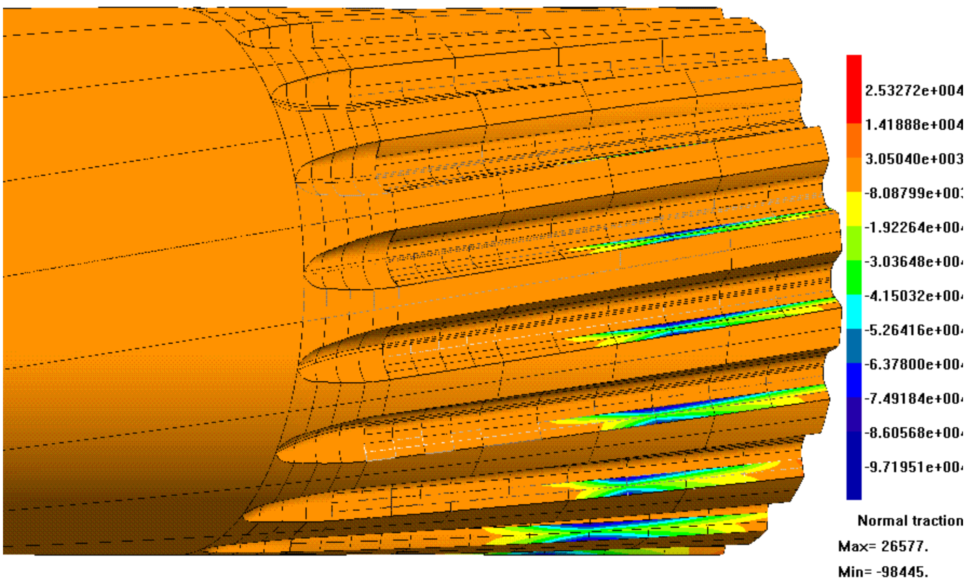


Fig 10 ***Detailed view of the male spline mesh in the full model. In this view the contact stresses are displayed***

COMPARISON WITH EXPERIMENTAL DATA

To provide an indication of the accuracy of the analysis, BE predicted stresses were compared with values derived from strain gauge and frozen stress analysis measurements.

A test spline was fitted with miniature strain gauges, which measured torsion, tooth bending and axial strains at a number of positions along the engaged length of the external teeth. The gauge length was small enough to avoid significant averaging of the measured strains. The test spline used was produced to a high degree of accuracy to minimise the effects of geometry errors. Gauges were duplicated at equivalent positions on a number of teeth to assess the variability due to geometry and gauge positioning errors. The test spline had modified contact geometry to improve load distribution. A multi-axis test rig was used to apply torque and axial load to the joint.

The photo-elastic models were produced by making moulds of the strain gauge test joint. Torsion and axial loads were applied using a dead-weight loading arrangement.

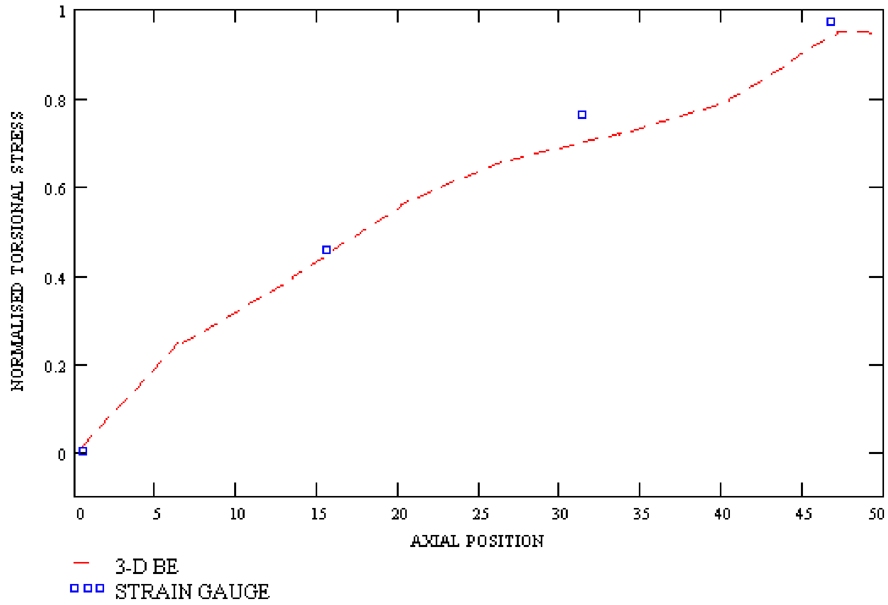


Fig 11 Comparison of 3d BEASY spline model Vs Strain Gauge Torsional Stresses

The BE analysis showed that the axial distribution of torsional tooth root stress is sensitive to the magnitude and axial distribution of contact stress. A comparison of the BE and measured values of torsional stress can therefore be used to provide an initial indication of the quality of the analysis. Figs. 11 and 12 show the BE analyses axial distribution of root torsional stress compared with stresses derived from the strain gauge and photo-elastic measurements at the spline design load. Good correlation is obtained in both cases.

Four strain gauges are inadequate to fully define the measured axial stress distribution, but as good agreement was obtained, the results are very encouraging. Comparisons made, Reference (3), at different loads and therefore at different contact conditions showed similarly good correlation and to some extent compensated for the small number of axial strain gauges. Good agreement was obtained between strain gauges at equivalent positions on different teeth.

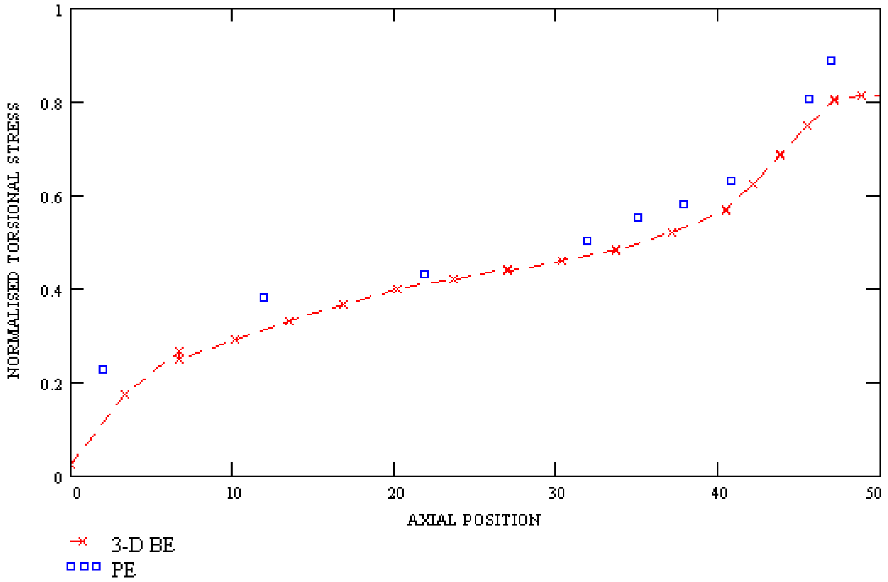


Fig 12 Comparison of BEASY model Vs Photoelastic Torsional Stresses

Due to different specific loading of the photo-elastic model, the contact conditions are very different to the steel spline and this is reflected in the results, and encouragingly, also predicted by the analysis. Averaged values are presented at Fig 12 because slight inaccuracies in the photo-elastic model and measurement resolution errors produced more scatter in the results than in the case of the strain gauge data.

Ideally, to fully validate the analysis, measurements of contact stress are also required; however, due to the difficulties involved this was outside the scope of the project. It is considered, however, that the degree of correlation obtained between the predicted and measured tooth root stresses, because of their sensitivity to contact stress distribution, shows that the analysis is providing a good representation of the overall spline stress state.

CONCLUSIONS

A technology has been developed capable of accurately predicting the behaviour of spline joints.
The software can simulate the contact stresses, slip and stick conditions and other data necessary to predict durability.
The model can predict the impact of manufacturing tolerances on the design and enable the user to investigate design options to achieve an optimum design.
A spline model generator has been developed to simplify the generation of the computer model.
Excellent agreement has been demonstrated with experimental data.

Acknowledgements

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REFERENCES

1. OHTE, S. “Finite element analysis of elastic contact problems”, *Bull. JSME*, 1973, **16**, 797-804
2. Pascoe. S. K, Mottershead, J. E., Hellen, T. K. A Comparison of finite element techniques for contact stress analysis. Modern Practice in stress and vibration analysis. Ed J.E. Mottershead, 1889 Pergamon pp 299-312
3. Comparison of Spline Boundary Element Analysis with Experimental Measurements. C McFarlane, T Hyde, E J Williams (To be published at IMechE Transmission Seminar, June 1999)
4. BEASY User Guide, Computational Mechanics, Ashurst Lodge Southampton, 1999