Technical Note: Comparison of boundary element and finite element methods for linear stress analysis — technical program results

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INTRODUCTION

Over the years, engineering organisations have become increasingly dedicated to improving productivity y through the use of high-speed computers, computer-aided design and manufacturing (CAD/CAM) techniques and computer-aided analysis methods such as the Finite Element Method. Despite the analytical method's substantial impact on productivity, three limiting factors persist: (1) the difficulty in identifying connectivity errors, particularly in 3D models, (2) the need to discretize the entire volume of a structure, and (3) the density of the mesh needed to obtain accurate surface stresses. A numerical technique which has a potential for eliminating these shortcomings is the Boundary Element Method. The purpose of this paper is to discuss the results of a technical program conducted at Hamilton Standard, comparing the Boundary Element and Finite Element Methods for two- and three-dimensional, linear structural analyses.

APPROACH

Three classes of linear, static, room-temperature stress applications were analyzed: 2D axisymmetric (body of revolution) stress, 2D plane stress, and 3D stress. The usefulness of the Boundary Element Method for these applications was determined in the categories of model generation time, analysis solution time, solution convergence and data reduction. The boundary element program BEASY*,[1] an in-house finite element program, and MSC/NASTRAN were used to perform this study.

MODEL DESCRIPTION

2D axisymmetric stress –

Both the barrel and cover of a typical actuator were constructed; see Fig. 1. The actuator barrel consists of 350 four noded quadrilateral elements and the cover consists of 529 elements. The applied boundary conditions are also shown in this figure. The boundary element models of this structure are shown in Fig. 2. Twenty-eight quadratic boundary elements represent the barrel and 64 elements define the cover.

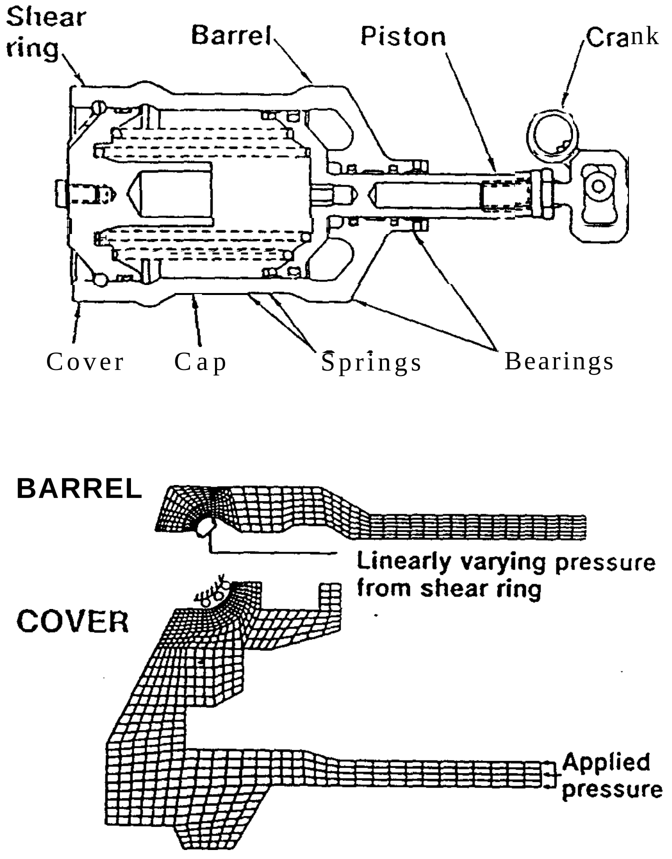

Figure 1. Actuator finite element models

2D plane stress –

An internal spur gear tooth was modeled for this case; see Fig. 3. The gear tooth model consists of 1,060 four noded elements. A point load was applied to the tooth pitch diameter and the tooth boundary was completely constrained. The boundary element model, shown in Fig. 4. consists of 41 quadratic elements.

3D stress –

One-eighth of a thick walled cylinder was selected for the 3D analysis; see Fig. 5. The finite element model shown consists of 240 linear strain elements and is subjected to an internal pressure of 100 psi. The boundary element model shown in Fig. 6 consists Of 12 quadratic discontinuous elements.

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* BEASY is a registered trade mark.

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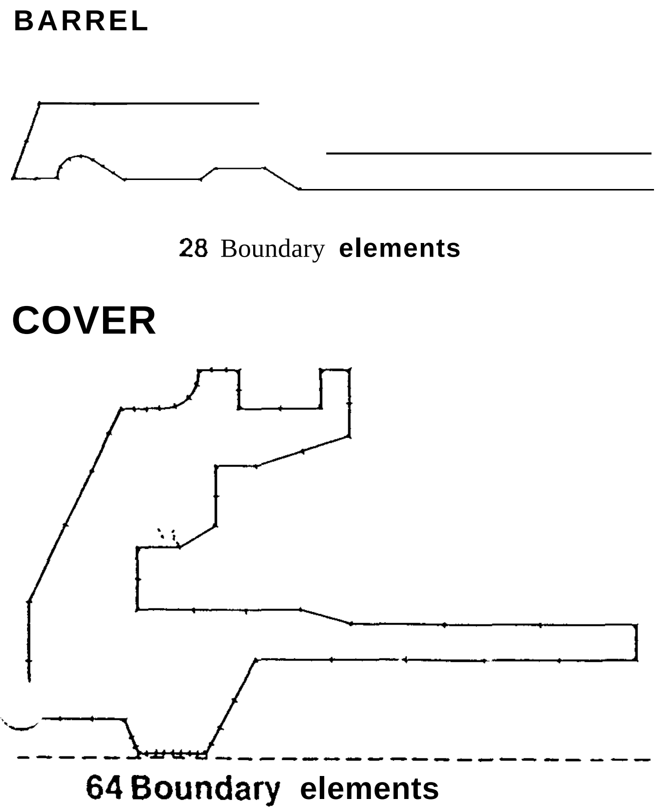


Figure 2. Actuator boundary element models

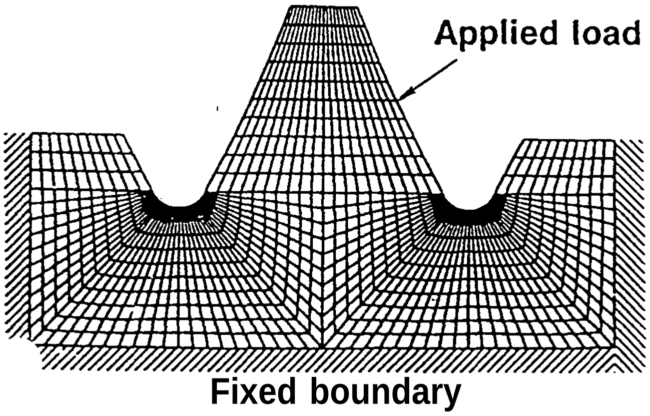


Figure 3. Finite element model of internal spur gear tooth

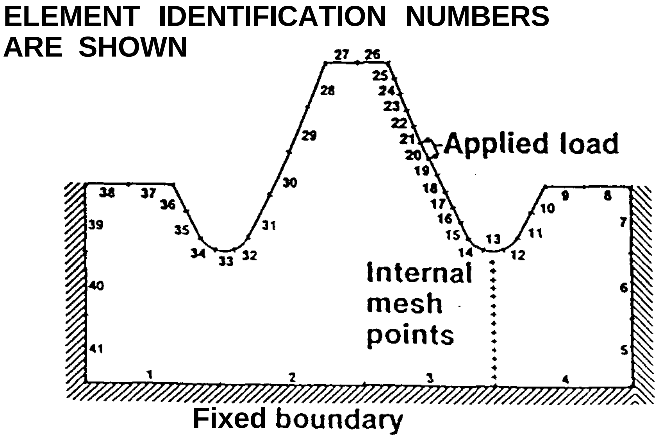


Figure 4. Boundary element model of internal spur gear tooth

PROGRAM RESULTS

Axisymmetric analysis –

A comparison of the boundary element and finite element surface stress for the actuator barrel is shown in Fig. 7. The maximum boundary element stress is 9% lower than the finite element solution. The boundary element stresses shown represent mesh point average values. When the difference between element stress contributions at a mesh point is 20% or greater, the BEASY program warns the analyst that a mesh refinement is required. Several boundary stress values, including the peak stress, were identified as having a mesh point average of 20% or greater.

Surface stress results for the cover are shown in Figs. 8 and 9. The boundary element model in the vicinity of

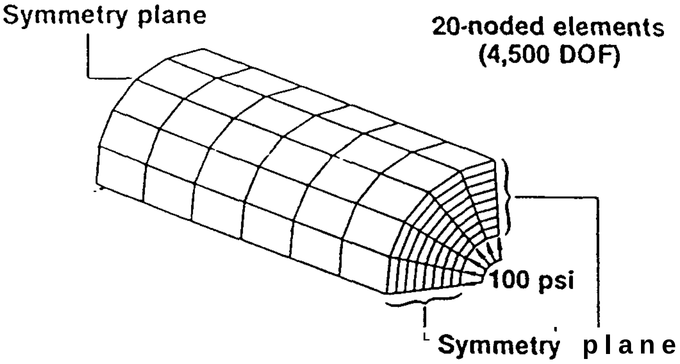


Figure 5. Finite element model of a thick wall cylinder

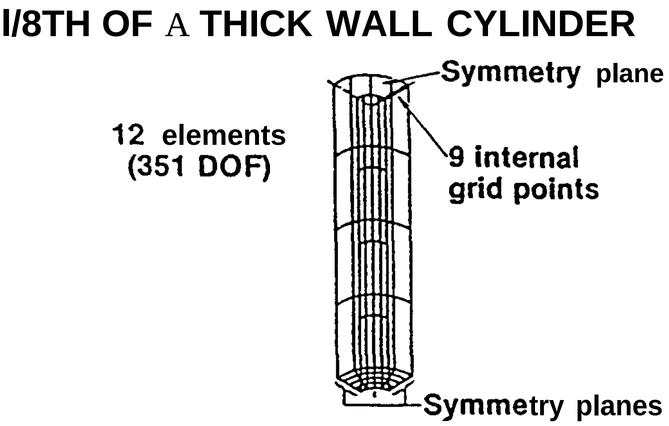


Figure 6. Boundary element cylinder model

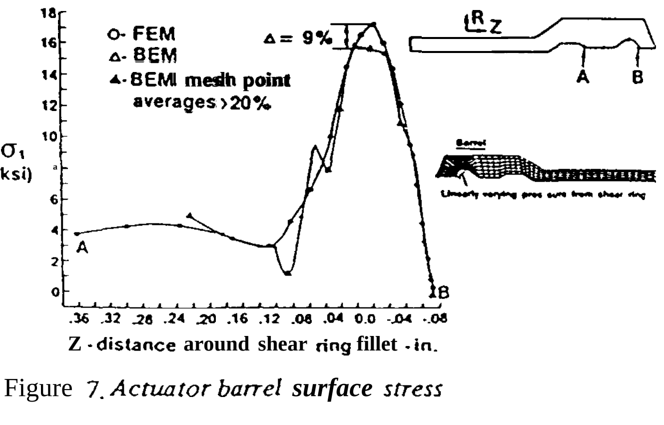


Figure 7. *Actuator barrel surface stress*

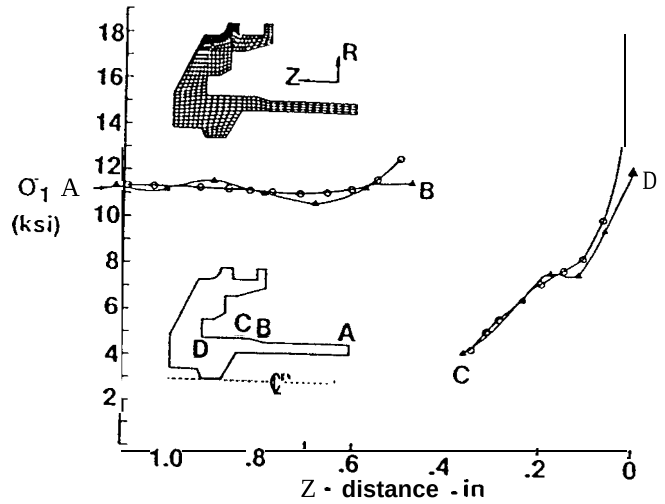


Figure 8. Actuator cover surface stress

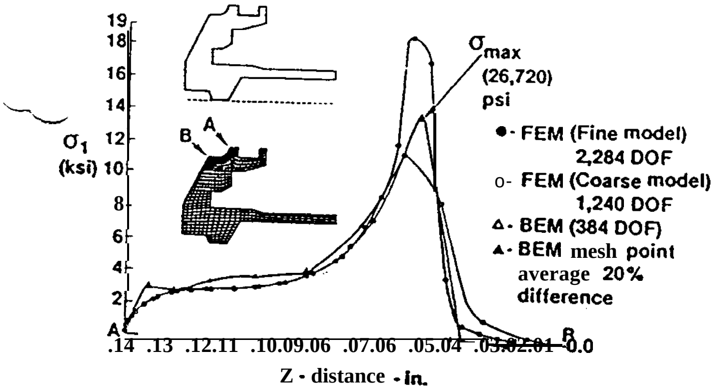


Figure 9. Actuator cover surface stress

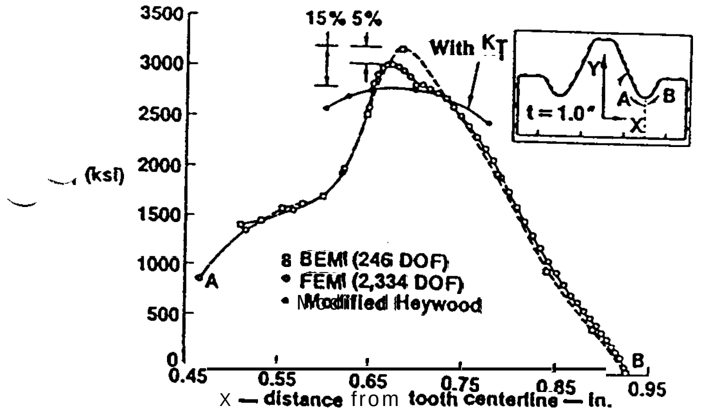


Figure 10. Tooth fillet surface stress

point D, shown in Fig. 8, would require a mesh refinement to obtain correlation with the finite element solution. To better determine the stresses between points A and B, shown in Fig. 9, a mesh refinement was made for both the finite element and boundary element models. While the refined boundary element peak stress (14 KSI) is substantially lower than the refined finite element peak stress (19 KSI), the boundary element mesh point average stress solution indicates that the true peak stress may be significantly higher (≈27 KSI).

Plane stress analysis –

Surface stresses, along with tooth fillet, are shown in Fig. 10. The boundary element peak stress is 5% higher than that of the finite element results. These results were also compared to a modified Heywood Method, described in ref. 2, which is based on photo-elastic tests. The peak boundary element stress is 15% higher than the strength of materials method.

A further comparison was made of the internal stresses obtained from both the Finite Element and Boundary

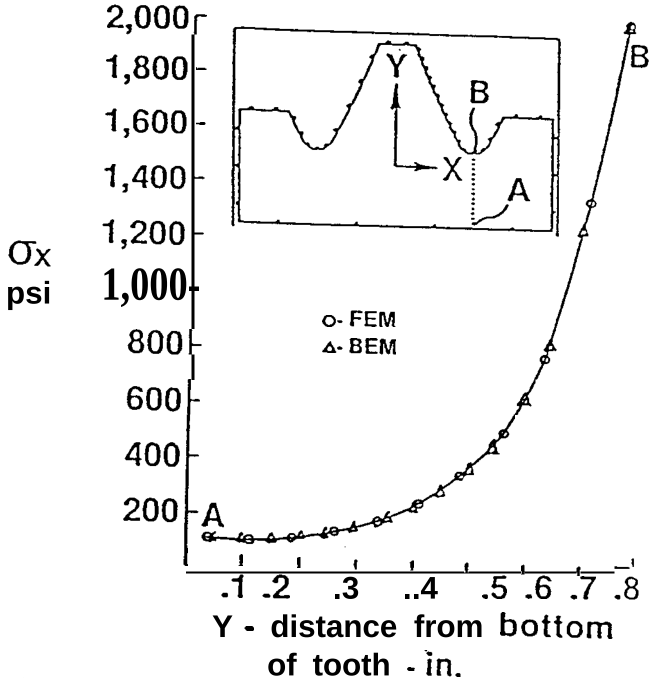


Figure 11

Analysis	Method	Model generation time (min.)	Lines of input	Solution time CPU (sec)
Internal spur gear tooth	FEM	60	130	18
	BEM	20	33	19
Actuator barrel	FEM	40	57	9
	BEM	10	36	12
Actuator cover	FEM	40	73	32
	BEM	15	61	33

Figure 12

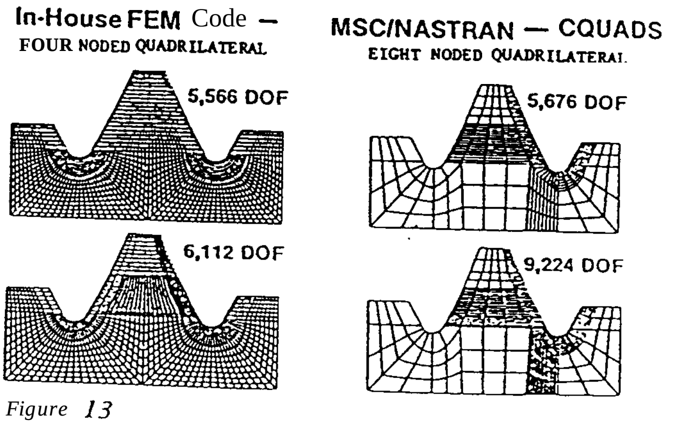
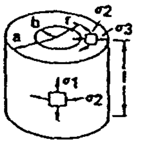


Figure 13

Code	Size (DOF)	CPU (sec)	σ max (psi)	Δ %
BEM-Beasy	246	19	3,210	-
	444	112	3,184	0.8
	528	164	3,193	0.3
FEM-in-house	2,334	18	3,053	
	5,566	67	3,374	10.5
	6,112	61	3,232	4.2
FEM-MSC/ Nastran	5,676	164	3,260	
	9,224	280	3,299	1.2

Figure 14



Computer Code	Analytical Method	Size DOF	Solution Time CPU (sec)	At r = b			
				σ_1 (psi)	σ_2 (psi)	σ_3 (psi)	Δb (in.)
—	Strength of materials (Roark)			0	133	−100	1.63E-7
BEASY	BEM	81	150	5	161	−147	1.86E-7
		162	95	5	161	−139	1.70E-7
		351	240	8	131	−105	1.69E-7
MSC/NASTRAN	FEM	579	20	14	141	−63	1.69E-7
		2664	184	5	136	−88	1.65E-7
		4500	331	3	134	−94	1.66E-7

figure 15

Element Methods; see Fig. 11. The two methods correlated very well.

2D modeling and solution time –

Modeling and computer solution times are summarized for the axisymmetric and the plane stress models in Fig. 12. All the analyses were performed on an IBM 3084 computer. The computer times listed are for the original models analyzed. The average modeling time using the BEASY preprocessor was approximately three times faster than the finite element modeling time. However, the Boundary Element Method, on the average, used approximately twice as much computer time than the Finite Element Method.

2D convergence –

Several additional models of the internal spur gear tooth were constructed to determine surface stress sensitivity to mesh size; see Fig. 13. The density of the finite element meshes was increased in the vicinity of the tooth fillet until the peak fillet stress did not change more than 5%. While the initial boundary element solution indicated convergence, two additional runs were made for the purpose of confirmation. Convergence results are shown in Fig. 14. The computer times of the converged finite element models range from 3 to 15 times higher than the initial boundary solution. This represents a more reasonable computer time comparison between the two methods than the previous 2D cases because it is based on the same degree of stress accuracy.

3D stress –

Stresses on the inner diameter of the cylinder are summarised in Fig. 15. A strength of materials calculation. obtained from ref. 2, is also tabulated. The finite element solution took 1.4 times longer to obtain the same accuracy in radial surface stress as the Boundary Element Method. However, the Finite Element Method took 130 cpu seconds less time to obtain a radial displacement, which is less than 5% of the theoretical value.

CONCLUSION

Use of the Boundary Element Method can favourably impact engineering productivity by significantly reducing the model generation and data reduction time. Since the Boundary Element Method does not require the discretization of the interior portion of a structure, fewer, if any, errors of geometry are made. Typically, determination of surface stresses is a primary objective in structural analysis. The Boundary Element Method produces surface stresses in a form which is more easily extracted and interpreted than the Finite Element Method. Also, the mesh point averaging technique, used in BEASY, indicates the accuracy of the solution, thereby, increasing the analyst's confidence in the results. The Boundary Element Method, unlike the Finite Element Method, calculates stresses and displacements directly, thus yielding the same order of accuracy for both. In principle, this means orders of magnitude less boundary elements are required than finite elements for determining stress. Use of the discontinuous boundary elements, as in the case of the 3D cylinder model, allows further reductions in mesh size and modeling time without reducing solution accuracy.

For the classes of structural applications analyzed, the boundary element computer solution time is comparable to the finite element time. Depending *on the* solution accuracy desired, the boundary element solution can take less time than the Finite Element Method.

One area which requires scrutiny is the analysis of highly complex three-dimensional structures which require a large number of elements. There may exist a point where the time saved in the generation of the model is offset by the computer solution time.

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