Some Applications of Boundary Element Method in Biomedicine

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INTRODUCTION

This paper describes some applications of the Boundary Element Method (BEM) [1][2] to simulate biomedical problems. The BEM has developed into a powerful tool for engineering analysis, but most applications of the new technique are in the fields of mechanical and aerospace engineering. Other important cases are for the study of electrical and fluid flow problems, and the range of applications is extending from day to day. The use of BEM for simulation of biomedical problems is, however, a comparatively new field, in spite of the many advantages that the technique presents versus more classical methods, such as finite elements (FEM).

The main attraction of BEM is that it only requires the definition of the surface of the body under study, while FEM needs the discretization of the whole body into a series of block-like elements. In addition, the BE technique is based on a mixed formulation, which implies that all the problem variables are found simultaneously on the body under consideration, so that all of them are computed with the same degree of accuracy. This property makes BEM especially attractive as standard FEM is based on a displacement - or potential - only formulation, in which errors are introduced when numerically carrying out derivatives of displacements - or potentials - to compute surface tractions - or fluxes - for instance. FEM results are neither in equilibrium on the boundary nor in the domain because, as already mentioned, the governing equations are solved in terms of displacements - or potentials. BEM results are in equilibrium inside the domain but not on the boundary where comparatively small errors are introduced by the interpolation functions used.

While the definition of the surface only of the body is an obvious advantage in model building, the mixed character of the BEM theory leads to very accurate results. Usually, even comparatively coarse meshes give good results in BEM, and this leads to the technique being renowned for its accuracy and reliability. Typical examples are stress concentration problems, where BEM gives accurate results with a relatively small number of elements, and for which many FEM solutions fail to converge even when the mesh is refined. For this reason BEM is frequently used in fracture mechanics applications to produce reliable stress intensity factors and to study more complex problems such as crack propagation.

BEM DISCRETIZATION

The most important feature of BEM is without doubt that it only requires discretization of the surface rather than the volume. This fact makes it easy to interface BEM codes with existing solid modellers and Computer Aided Design systems. The reduced modelling time is particularly important in the design process which normally involves a series of modifications and redesigns.

BE models are simple to create even in three dimensional cases, as the mesh is defined only on the external surface. The discretization process is facilitated by the use of discontinuous elements, which are unique to BEM. In finite elements continuity of the displacements (or potentials) between elements is required, but this is not needed in BEM a-s the technique is based on a mixed formulation. Well written B EM codes [3] [4] take advantage of this property, using discontinuous elements in some regions as required and continuous ones elsewhere as the latter reduce the number of degrees of freedom required to solve the problem. Figure 1 illustrates an application of the use of discontinuous elements. The figure represents one eighth of a cylinder wit h a perforation through its cent re. As can be seen, the elements have been concentrated in regions where high gradients are expected and the mesh refinement can be achieved by changing rapidly from smaller to larger elements without the need to satisfy continuity. In addition, the elements are not required to be continuous at the edges which simplifies the whole process of designing a mesh. Consequently, mesh design can readily be automated, so that "elements" become effectively transparent to the user. It is also worth noting that elements are not needed on the three planes of symmetry. For further details of symmetry problems the reader is directed to references [2] and [5].

Boundary elements are nowadays frequently used by the automotive and aerospace industries as they are simple to implement as part of the Computer Aided Design process. Engineers prefer them to FEM because they can easily model complex 3-D structures.

ACCURACY OF THE RESULTS

One of the motivations for the use of boundary elements in engineering practice is the high degree of accuracy and reliability of the results. The accuracy is a consequence of the use of an analytical solution (i.e. the fundamental solution) as a weighting function, and the fact that the formulation is of a mixed character and produces displacements and stresses within the same degree of precision.

The errors in boundary elements are due to idealizations, approximations and numerical implementation of the technique, in addition to the round off and precision errors associated with repeated computations. The latter can become an important factor in boundary element software if care is not taken to condition the equations properly.

Idealization errors derive from how well the numerical model represents the real problem and in this regard the BEM is is also generally affected by variations in loads and boundary conditions, although it is less sensitive to mesh refinement than FEM. The latter property produces much better behaviour of the BEM results and a general robustness of the technique.

Approximation errors are those associated with having insufficient elements to describe the problem to achieve convergence. As expected, increasing the number or order of elements will reduce this error.

Implementation errors in BEM are very improtant as the technique requires the evaluation of singular or nearly singular integrals amongst others. They are mainly associated with the schemes used to compute the numerical equations. Recently, special schemes have now been developed for boundary element computations which have helped to reduce these errors considerably.

CASE STUDIES

A few case studies will now be presented to illustrate some applications of the BEM in biomedicine. There is a whole range of other problems which will not be presented here, such as tomography studies, prostheses, bone and dental mechanics, blood circulation, encephalographical studies and many others.

The examples presented here are

i) Model of cornea-scleral shell under internal pressure

ii) Simulation of stresses in a heart

iii) Aortic valve stress analysis

i) <u>Model of Cornea-scleral shell</u>

This model has been studied by Ghista, Kobayashi *et al.* using finite elements [6] aimed at prediction of the intracoacular pressure from the indentation of the cornea surface in tonometry procedures. The model is shown in Figure 2. The values of the modulus of elasticity and Poisson's ratios for the different regions were as follows.

$$\text{STROMA } E = 2.0 \text{ N/mm}^2 \; ; \; = 0.45$$

$$\text{SCLERA } E = 5.5 \text{ N/mm}^2 \; ; \; = 0.45$$

The modulus of elasticity of the limbus is assumed to vary from 2. to 5.5 N/mm. The analysis was carried out linearly assuming that the vitreous humour has a modulus of $E = 0.02$ N/mm.

Under these conditions the eye has been analysed using a BEM mesh as described in Figure 3. Notice that the limbus has been discretized in a series of 5 homogeneous regions to take into consideration better the variation of its properties. The internal pressure in the eye has been represented as equivalent thermal effects and this gives a fluid pressure of around 17 mm Hg.

Under these conditions the distribution of stresses in the eye are presented in Figure 4 and can be seen in more detail for the limbus region in Figure 5. It is now possible to represent the level of pressure against the front of the eye for instance using prescribed displacements in the region of the stroma, and this will give a diagram of pressure versus the radial distance which can be used to study Goldman application tonometry. The results, although linear, will give an indication of the pressures inside the eye which can be used to study the process and improve the experimental apparatus.

ii) <u>Simulation of Stresses in a Heart</u>

Next, an axisymmetrical model of a heart has been studied following the FEM results presented in reference [6]. The shape of the idealized heart used in both cases is shown in Figure 6 together with its discretization into boundary elements. As a first attempt the material was assumed to be isotropic with $E = 0.006$ N/mm and $= 0.45$. The model was loaded under internal pressure of 240 mm of water and the resulting von Mises stresses plotted in Figure 7.

It was pointed out by Ghista *et al.* [6] that the isotropic model yields values of the deformed linear radius approximately 8% less and stress levels that are double those of the anisotropic case. This points out the importance of anisotropy and the need to cent inue this research taking into account more realistic material properties.

iii) Stress Analysis of an Aortic Valve

This example concerns stress analysis of a human aortic valve leaflet using BEM which can be applied towards designing a prosthesis. The shape of the valve in Figure 8 has been taken from reference [7] where the contour map of the leaflets has been presented. Due to symmetry one needs to consider only the shape of half of each leaflet which is represented as shown in Figure 9, where the BEM discretization can also be seen. The shape was then assumed to have symmetry conditions along one edge and be simply supported along the other two when subjected to an internal pressure.

Figure 10 shows the stress patterns on the surface of the leaflet which appear to be similar to those of reference [7] with the main resistance to the applied pressure in the circumferential direction and showing the absence of compressive forces.

This type of study can result in better ways of analysing sort ic valve prostheses to ensure their structural integrity.

ADVANTAGES OF BEM

The main advantages of the BEM are:

- Bem integrates over the boundary and hence the user needs to define only the surface of the component under analysis. This results in man-time improvement typically in the region of one order of magnitude over FEM. This is important as FEM data preparation presently accounts for around 85% of the cost of performing an analysis.
- A BEM model is simple to change as any modifications are purely local. This is particularly useful in a research or design environment as it allows alterations to be carried out quickly.
- The BEM solutions are not so sensitive to mesh grading as those in FEM and hence the results are more reliable.
- The results of BEM are of superior accuracy to FEM results. This is particularly true for problems such as stress concentration and fatigue where accuracy is of prime importance. It is also comparatively easy to learn how to use BEM systems. They require less training and skills than FEM codes.

CONCLUSIONS

The BEM provides an excellent tool for the analysis of many biomedical systems. The technique possesses the advantages of simple model generation, easy to understand requirements, high accuracy, relative insensitivity to mesh refinements and the ability to accurately model the most difficult stress concentration problems.

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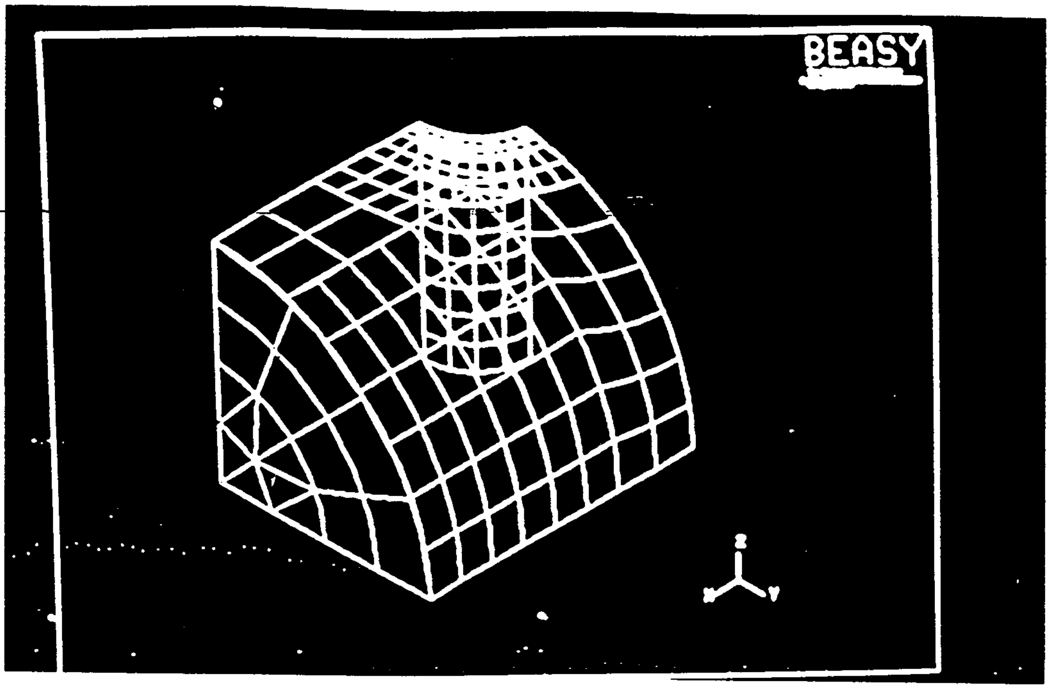


Figure 1: One eighth of a cylinder with a cylindrical perforation

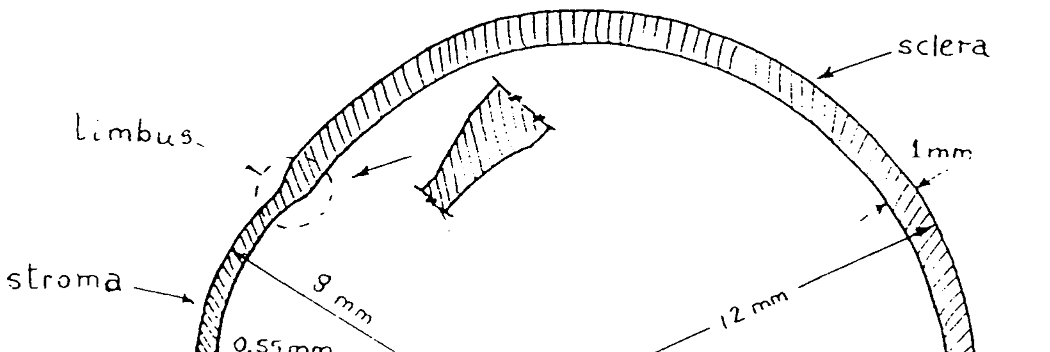


Figure 2:Model of idealized human eye

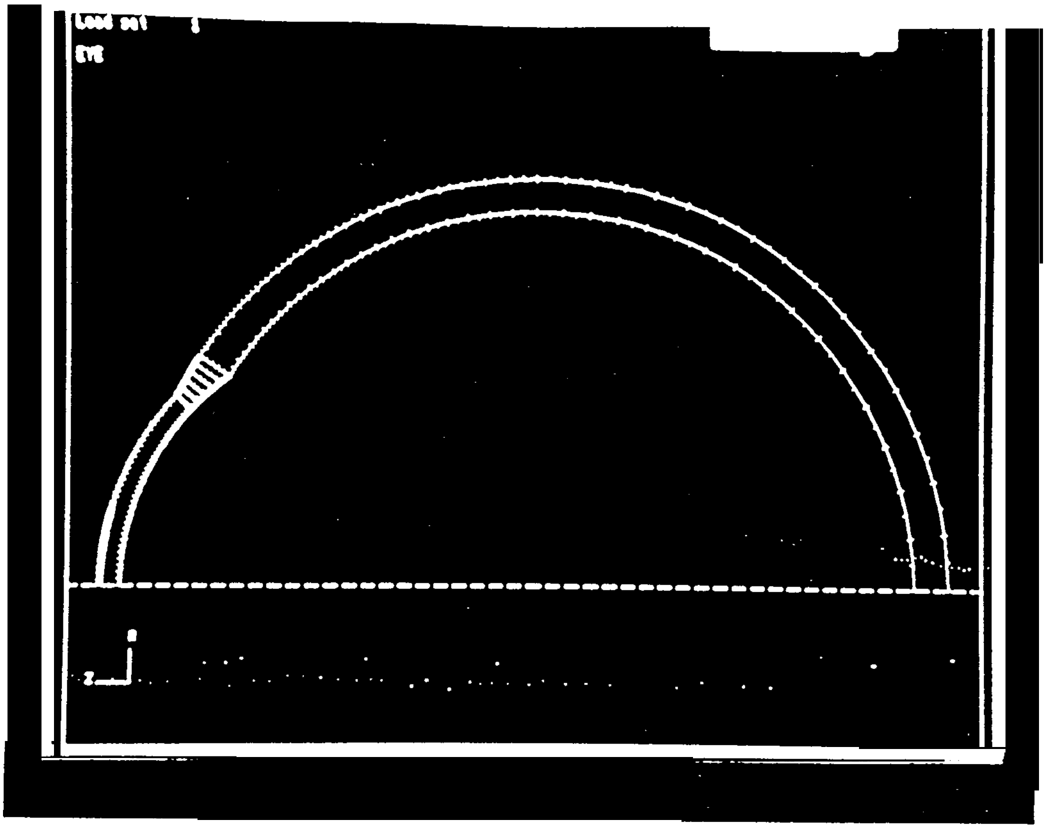
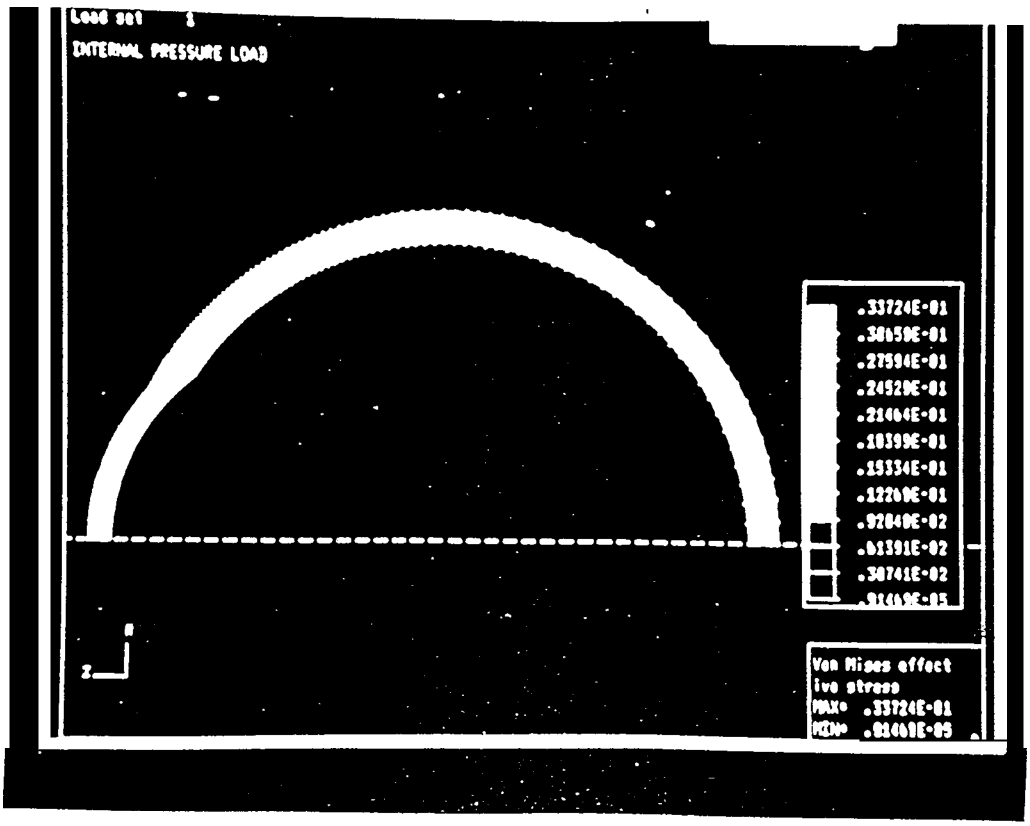


Figure 3: Boundary Element discretization of cornea scleral shell



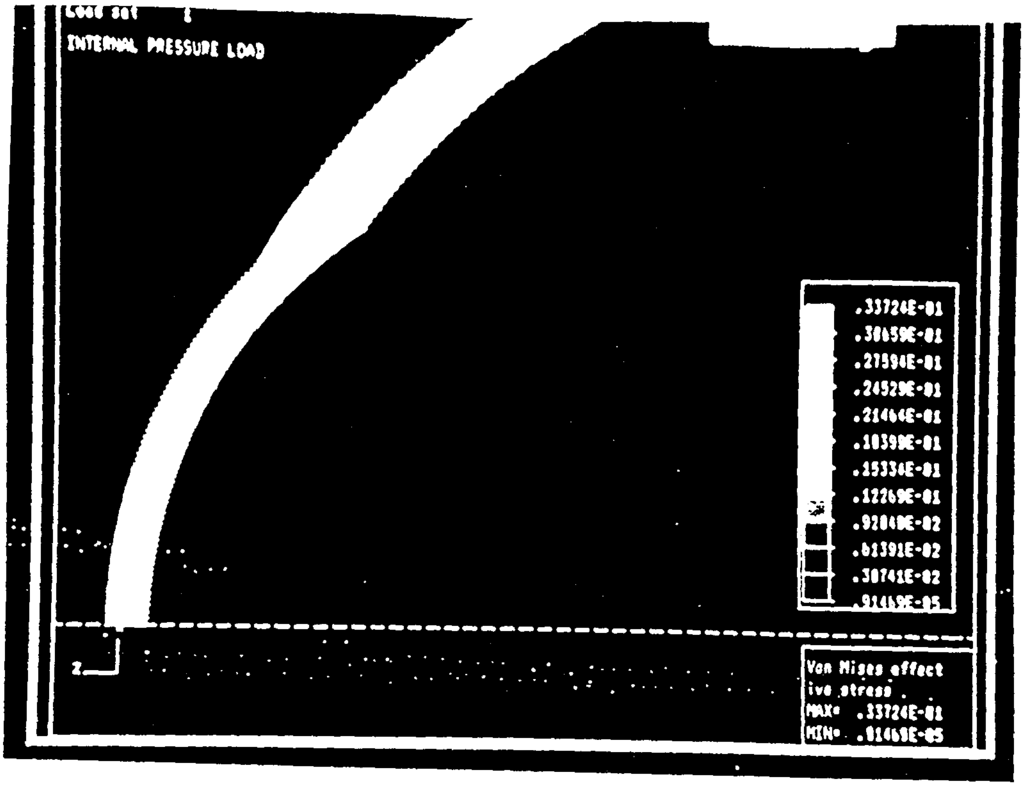
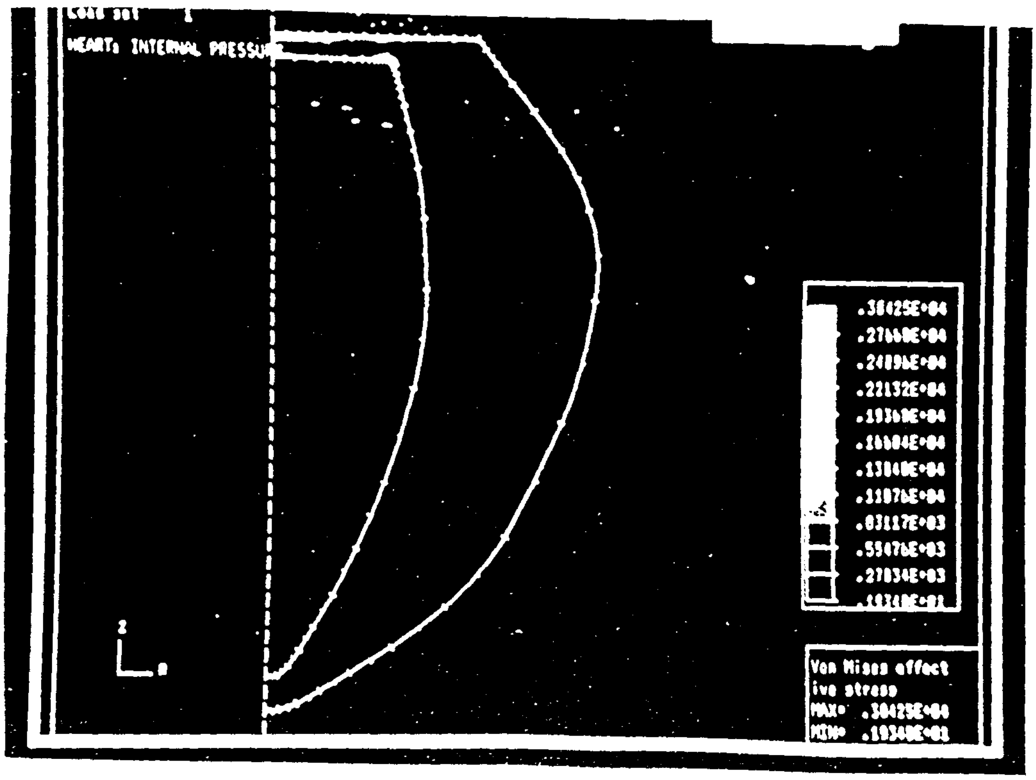
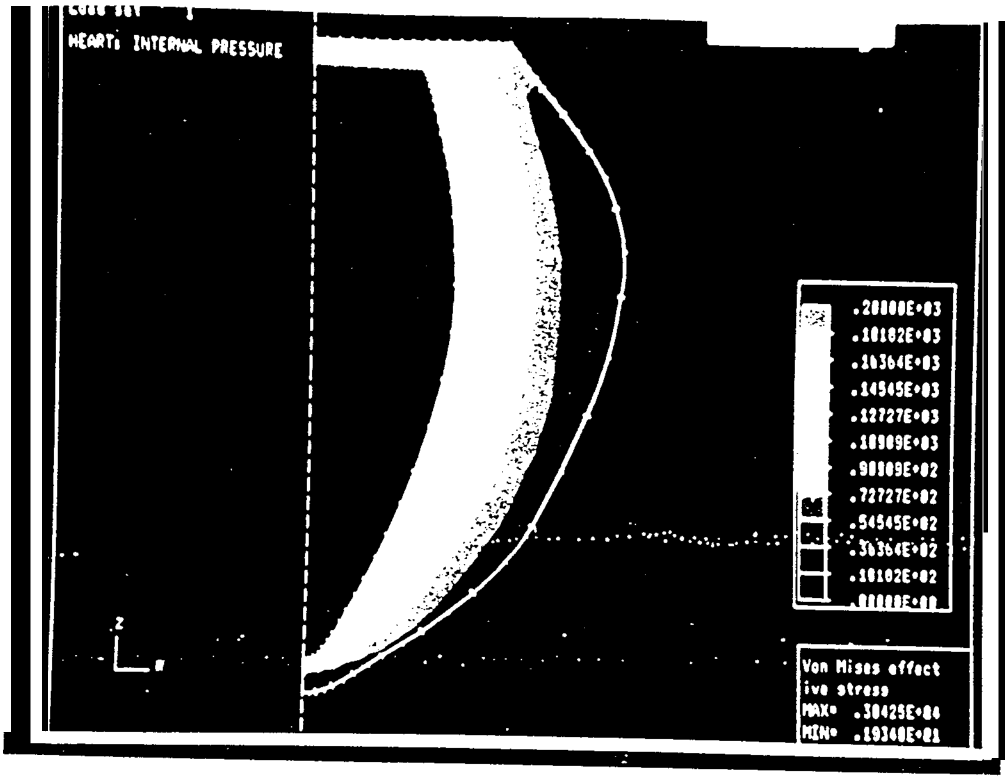


Figure 5: Effective stress distribution in region of the [illegible]

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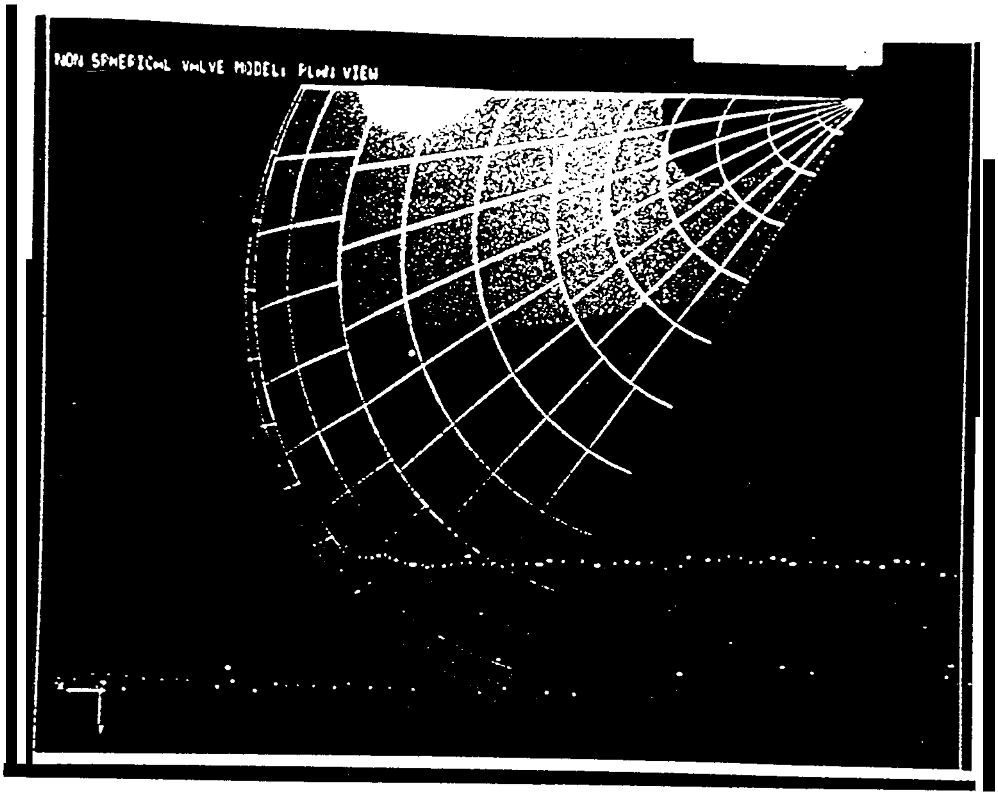

Figure 9: Discretisation of half a leaflet of an aortic valve