Predicting Fatigue Crack Growth in Complex Components

by A.C. Neves, S.M. Niku, R.A. Adey
Computational Mechanics BEASY, Ashurst Lodge, Ashurst, Southampton, SO40 7AA, U.K.

Abstract

Catastrophic failure of engineering structures is caused by cracks that extend beyond a safe size. Cracks are present to some extent in all structures and can lead to failure or decrease in structural strength if they grow. This paper described an automatic procedure for predicting the fatigue crack growth in two and three dimensional structures and the key data for fracture mechanics based design: the stress intensity factors.

The procedure is implemented in a general purpose engineering analysis system BEASY. The system uses a dual boundary element technique to simplify the modelling of the cracks and to provide highly accurate stress intensity factors. Applications are presented for single and multiple cracks.

1.0 Introduction

The cost to industry of fracture was recently estimated in a report of the US Department of Commerce entitled "The economic effect of fracture in the United States". In this report it estimated that the cost of fracture was $119 billion dollars per year (4% of gross national product).

It further estimated that approximately one third could be saved through the use of current fracture control technology and a further 25% could be saved through fracture related research. Therefore the annual cost of fracture could be halved by the application of better design tools based on fracture mechanics technology.

The combination of the easy to use and apply boundary element approach in BEASY and the new fracture growth tools incorporated in the system are aimed to satisfy this goal.

Catastrophic fracture failure of engineering structures is caused by cracks that extend beyond a safe size. Cracks, present to some extent in all structures, either as a result of manufacturing fabrication defects or localised damage in service, may grow by mechanisms such as fatigue, stress-corrosion or creep. The crack growth leads to a decrease in the structural strength. Thus, when the service loading cannot be sustained by the current residual strength, fracture occurs leading to the failure of the structure. Fracture, the final catastrophic event that takes place very rapidly, is preceded by crack growth which develops slowly during normal service conditions, mainly by fatigue due to cyclic loading.

Damage tolerance assessment is a procedure that defines whether a crack can be sustained safely during the projected service life of the structure. Damage tolerance assessment is, therefore, required as a basis for any fracture control plan, generating the following information, upon which fracture control decisions can be made:

The effect of cracks on the structural residual strength, leading to the evaluation of their maximum permissible size.

The cracks growth as a function of time, leading to the evaluation of the life of the cracks to reach their maximum permissible size, from which the safe operational life of the structure is defined.

Linear elastic fracture mechanics can be used in damage tolerance analyses to describe the behaviour of cracks. The fundamental assumption of linear elastic fracture mechanics is that the crack behaviour is determined solely by the values of the stress intensity factors which are a function of the applied load and the geometry of the cracked structure. The stress intensity factors, thus play a fundamental role in linear elastic fracture mechanics applications.

Crack-growth processes are simulated with an incremental crack-extension analysis. For each increment of the crack extension, a stress analysis is carried out and the stress intensity factors are evaluated. The crack path, predicted on an incremental basis, is computed by a criterion defined in terms of the stress intensity factors.

The boundary element method is well established as a powerful solution tool for fracture mechanics (see Aliabadi & Rooke [1]). The reason for its success is the boundary only representation, the high accuracy and the methods ability to represent the high stress fields near the crack front.

The theoretical foundation of the dual boundary elements and crack growth algorithm follows closely the work of Portela and Aliabadi [2] for 2D; and Mi and Aliabadi [3] for 3D. The authors will not repeat the theoretical basis but would refer the reader to references [2] and [3] for the detailed basis of the crack growth and dual boundary elements and to Brebbia and Dominguez [4] for a description of the boundary element method. The fatigue life predictions are evaluated using the generalizes formula presented in NASA/FLAGRO 2.0 [5].

In this paper the implementation within the BEASY boundary element system is described and application presented showing the wide range of applications possible.

2.0 Simulation Strategy

A crack can be represented in a boundary element model using two main approaches. The traditional approach requires the user to define a zone boundary along the crack surface and continue this through the body of the components being studied. This can be clearly seen in Fig. 1b where the problem is split into two zones and the edge crack is extended by a zone interface (shown as a dotted line) across to another external boundary. This technique while not requiring any special theoretical development places on extra burden on the user.

The second approach recently developed is to use dual boundary elements to represent the crack. Fig. 1a shows the model of the edge crack using this approach. In this case the modeling is extremely simple and economical. The crack is represented by two elements occupying the same physical location, each element representing a face of the crack.

Multiple cracks can be easily defined by simply specifying the initial crack size. Crack merging can also be automatically modelled

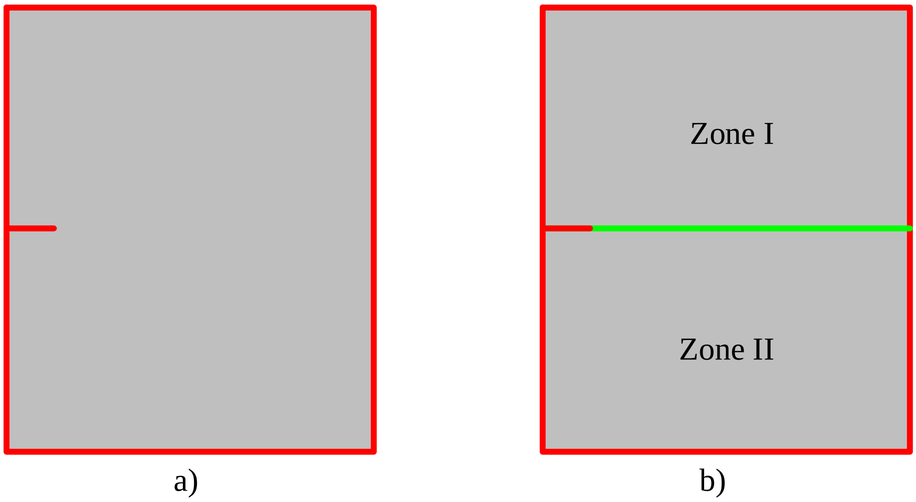

FIGURE 1. Boundary element models of edge crack. a)Shows the crack modeled using dual boundary elements. b) Shows the crack modeled using two zones.

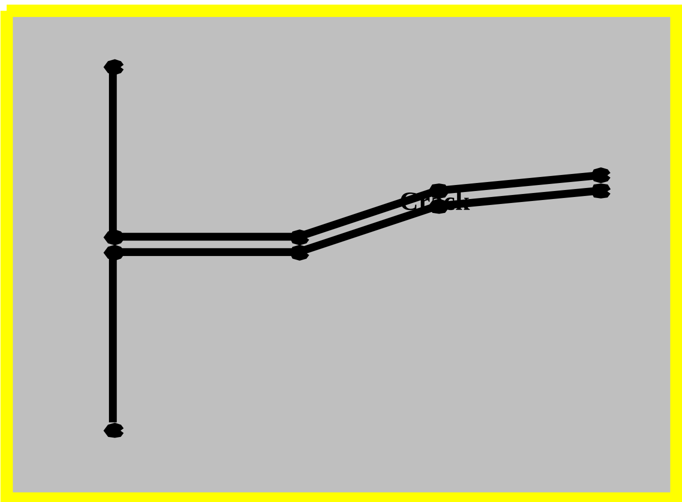

FIGURE 2. Dual boundary element representation of the crack.

2.1 Stress Intensity Factors

The system automatically computes the stress intensity factor using both J integral (2D only) and crack openning displacement (COD)formula. These methods can be used for computing Modes I, II and III intensity factors for edged and embedded cracks.

The J integral path is automatically computed by the software and follows the crack tip as it grows into the material.

2.2 Incremental Crack-extension Analysis

The incremental crack-extension analysis assumes a piece-wise linear discretisation of the unknown crack path. For each increment of the crack extension, the dual boundary element method is applied to carry out a stress analysis of the cracked structure and either the J-integral or the COD formula is the technique used for the evaluation of the stress intensity factors. The steps of this basic computational cycle, repeatedly executed for any number of crack-extension increments, are summarised as follows:

- Carry out a stress analysis of the structure
- Compute the stress intensity factors
- Compute the direction of the crack-extension increment
- Extend the crack one increment along the direction computed in the previous step
- Repeat all the above steps sequentially until a specified number of crack-extension increments is reached

For the sake of simplicity, the increment of crack extension is discretised with a fixed number of new boundary elements. In order to avoid numerical problems, concerned with the relative size of neighbouring elements, the crack increment length is kept between convenient limiting bounds, defined in terms of the size of the crack-tip element. Apart from this constraint, the length of the crack extension increment may be defined as the result of a compromise between accuracy and computational cost; the smaller the crack increment the more accurate and expensive is the analysis.

The results obtained from an incremental crack-extension analysis are a crack path diagram and diagrams of the stress intensity factor variation along the crack path.

2.3 Crack-extension Criterion

Several criteria have been proposed to describe the mixed-mode crack growth. Among them, the most commonly used are the maximum principal stress and the minimum strain energy density.

The maximum principal stress criterion formulated by Erdogan & Sih [7] postulates that the growth of the crack will occur in a direction perpendicular to the maximum principal stress. Thus, the local crack-growth direction is determined by the condition that the local shear stress is zero.In practice this requirement gives a unique direction irrespective of the length of the crack extension increment. Therefore the procedure adopted in the system is to use a predictor corrector technique to ensure the crack path is unique and independent of the crack extension increment used.

The minimum strain energy density formulated by Sih [8] is based on the hypotheses:

- The direction of crack propagation at any point along the crack front is toward the region with the minimum value of strain energy density factor S as compared with other regions on the same spherical surface surrounding the point.
- Crack extension occurs when the strain energy density factor in the region $S=S_{min}$ reaches a critical value S_{cr}.
- The length r_0 of the initial crack extension is assumed to be proportional to S_{min} such that S_{min}/r_0 remains constant along the new crack front.

In 2D applications, the system uses as default the maximum principal stress while in 3D cases only the minimum strain energy density is adopted.

2.4 Crack Growth Relationship For Fatigue

A number of crack growth laws have been developed relating the rate of growth of the crack to the stress intensity factor. BEASY evaluates the fatigue life prediction using the generalized NASGRO 2.0 equation [5]:

$$\frac{da}{dN} = \frac{C \cdot (1-f)^n \cdot \Delta K^n \cdot \left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{(1-R)^n \cdot \left(1 - \frac{\Delta K}{(1-R)K_c}\right)^q} \tag{1}$$

It should be noted that equation (1) may be reduced to the Paris equation [6] by setting the parameters p and q equal to zero and not considering the effect of crack closure, *i.e.* $f=R$ for $0<R<1$. The Paris equation is given by:

$$\frac{da}{dN} = C \cdot \Delta K^n \tag{2}$$

The database of fatigue material constants supplied by NASA and ESA has been integrated with the crack growth algorithm and this can be used to select standard data; or the user can define alternative database containing the material constants.

3.0 Numerical Validation

The accurate prediction of the crack growth is highly dependent upon the accuracy of the stress intensity factors. Any minor innaccuracy in these values is accumulated during crack growth process which can lead to a completely unrealistic prediction.

Therefore considerable care has been taken in selecting and implementing the techniques used to compute the SIF values. A large number of tests have been carried out to verify the accuracy and reliability of the software for simple modes and mixed mode cracks.

Comparisons of the stress intensity factors have been made with a number of published test solutions and the accuracy found to be very high. For single mode and mixed mode cracks the solution has been found to be almost exact.

3.1 Gear Tooth

The growth of a crack from the root of a gear is investigated. The problem was originally studied by Flasker and Pehan [9]. In this example, the NASGRO equation is used with stress ratio $R=0.5$ and total number of cycles $N=10^6$. The curve fitting constants of the titanium alloy (code P1CA13WA1) was taken from the database file NASMFM. Crack closure effect was not taken into consideration. The crack growth until failure,i.e.,K_{max} exceeds the fracture toughness (K_c) of the material.

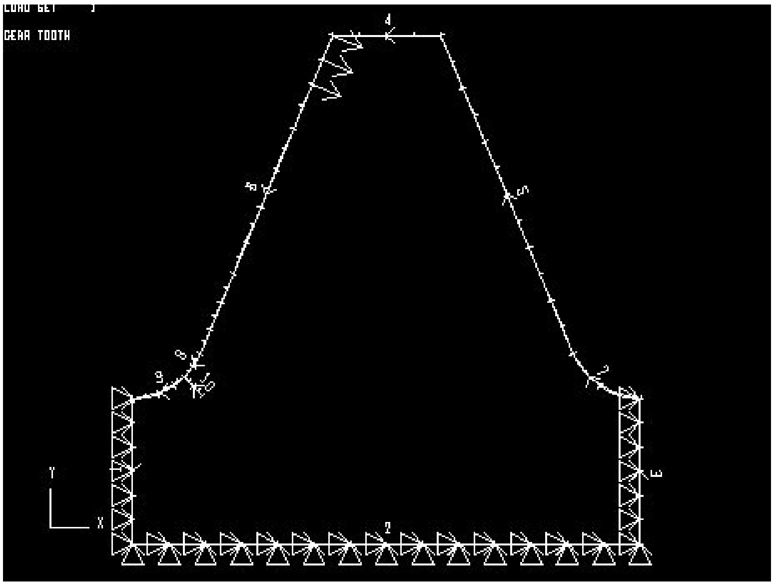

FIGURE 3. Complete model geometry of the gear tooth with the applied loading.

Figure 4shows the deformed shape of the gear tooth and the crack path. This compares well with experimental results and manual finite element calculations reported by [9]

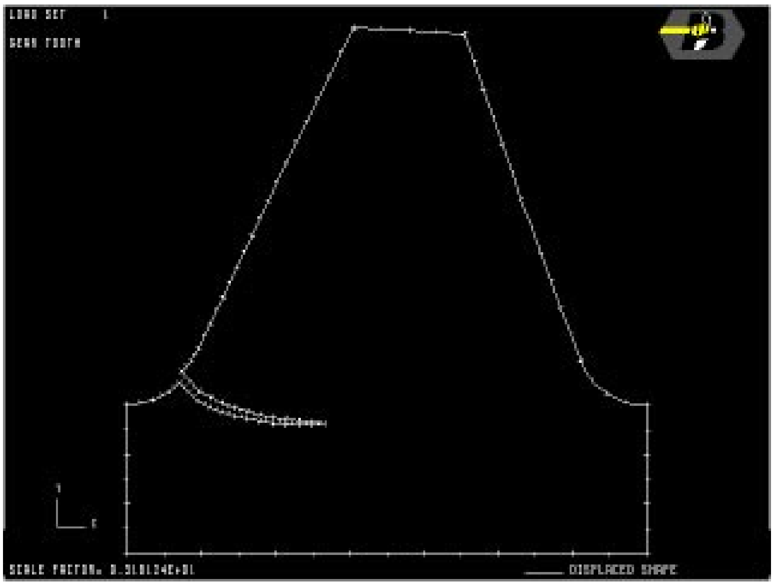

FIGURE 4. Deformed shape of the gear tooth.

Figure 5 shows the stress intensity factors varying for each crack growth increment. One can notice that at the final increment, the effective stress intensity factor is greater than the fracture toughness. The extension of the crack per number of cycle loading is given in Figure 6.

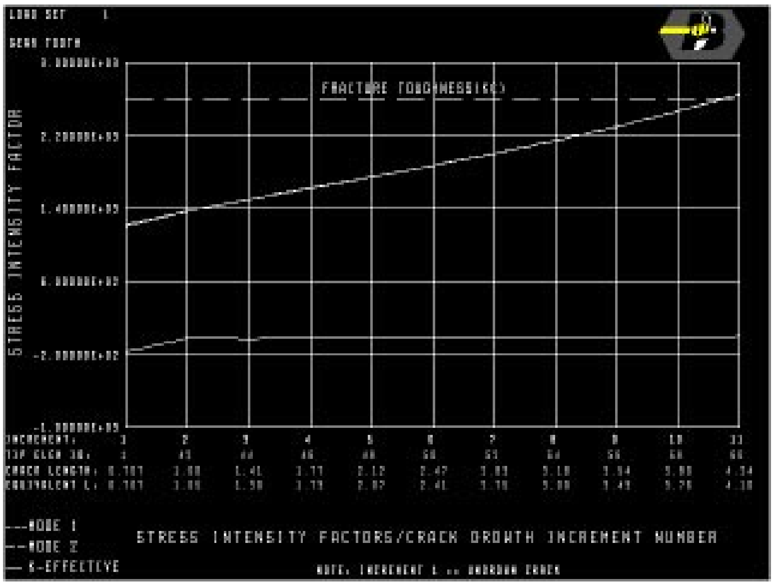

FIGURE 5. Stress intensity factors x crack growth for the gear tooth.

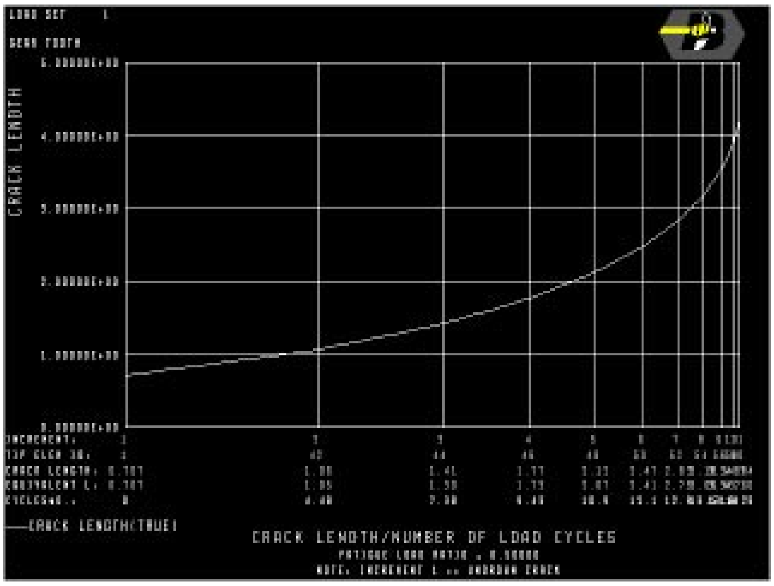

FIGURE 6. Crack length x Number of load cycle .

3.2 Multiple crack growth

In this application, the effect of cracks initiating from the rivet holes of an aircraft panel is investigated. The plate is fixed at the bottom side and a normal traction applied at the top. The complete crack path and history can be predicted for multiple site damage studies. The successful completion of this analysis provides the data required to predict the total fatigue life *before* the component goes into service. The safe design life can be stablished by use of suitable safaty factors.

Compared with other techniques, this computer simulation provides much more information much quicker and much cheaper. With any technique, a single prediction does not represent the final solution but is merely the starting point for sensitivity studies. The flexibility, accuracy and ability of this computer based techique to model multiple site damage in complex strucures is the authors believe a major advance.

As the cracks grow the software automatically adjust the rate of crack growth for each crack, based on the crack growth law. This can be seen in this example where the cracks near the edge grow much slower than the others.

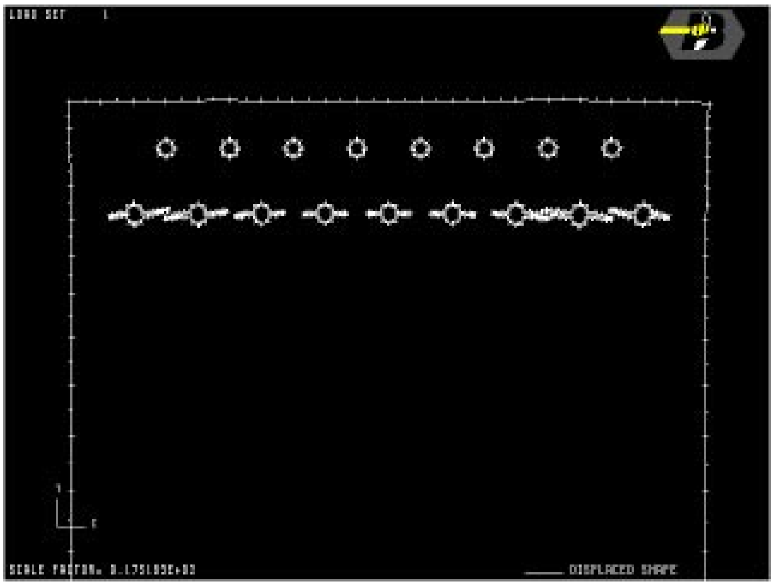

FIGURE 7. Aircraft panel with rivet holes.

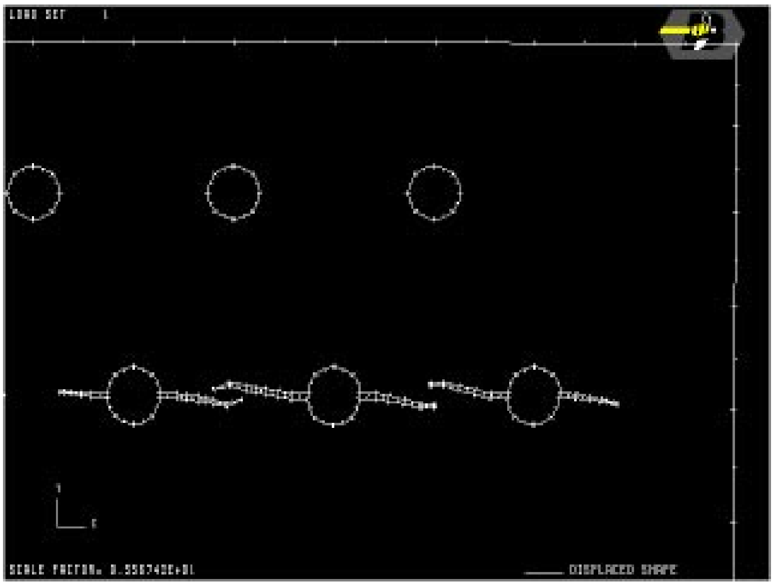

FIGURE 8. Detail of the cracks in the rivet holes.

3.3 Embedded Elliptical Crack

A mixed mode 3D crack growth problem is considered. A 45^o inclined elliptical crack of major semi-axis a and minor semi-axis b is embedded in a cylindrical bar of radius R and height h as shown in Figure 9. The ratios of these sizes are $R/a=10$, $h/R=6$ and $b/a=0.5$. Material constants are Young's modulus $E=100\ kN.mm^{-2}$ and Poisson's ratio $\nu=0.3$. A fatigue cracking process is generating by a constant amplitude cyclic tensile loading applied on the ends of the bar with stress ratio $R=0$ and $\sigma_{max}=100\ N.mm^{-2}$. Material constants used in the Paris law are $C=1.5463x10^{-16}$ and $m=3.88$.

A total of four steps of increment were performed. The crack path after the last increment are shown in Figure 10 and it is in good agreement with reference [3].

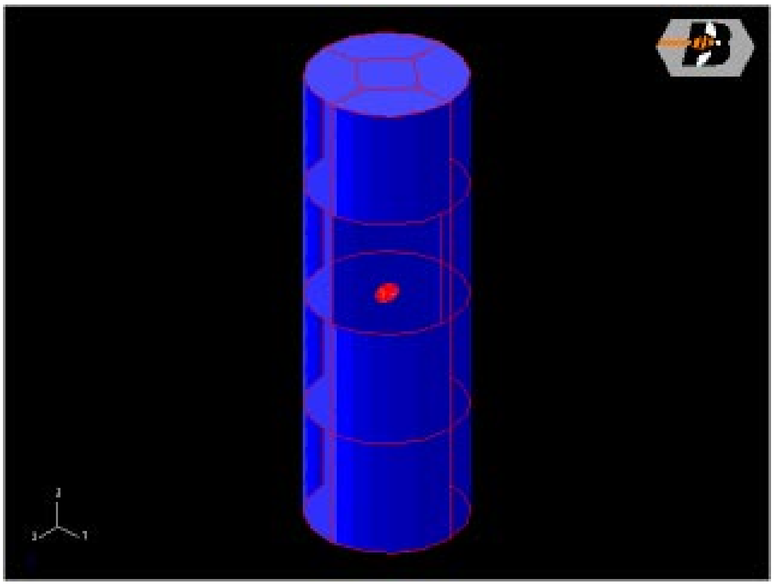

FIGURE 9. Cylinder with embedded elliptical crack.

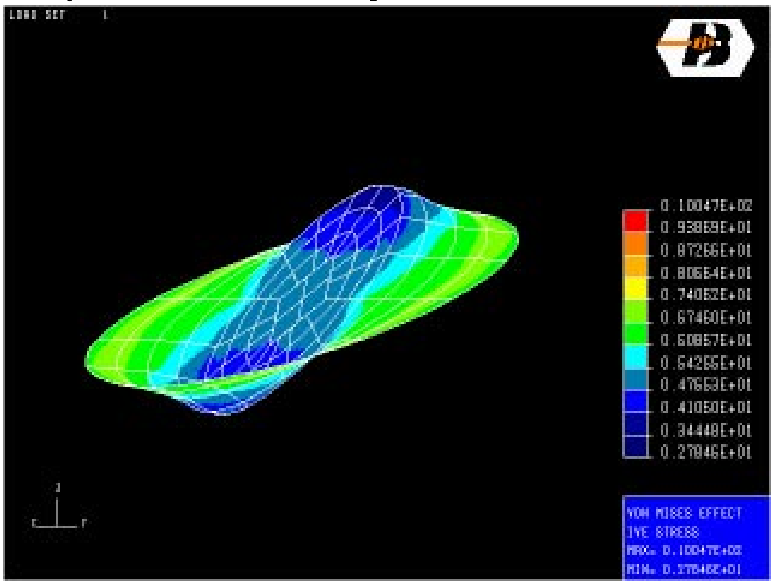

FIGURE 10. Von Mises stresses and crack growth path of the inclined elliptical crack

4.0 Conclusion

The paper describes a powerful system for the prediction of fracture data and fatigue crack growth in two and three dimensional structures. The techniques have shown high accuracy and the ability to automatically predict the growth of cracks. The system is particularly powerful because of the very simple modeling and complete lack of meshing problems associated with FEM solution of these problems.

Acknowledgement

The authors acknowledge the support of Dr Aliabadi during this work.

This work was partly funded under Project MASS of the European Community. Partners in the project include Computational Mechanics BEASY, Microparts, Technical University of Berlin and Wessex Institute of Technology.

Additional support was provided by the European Space Agency.

References

1. Aliabadi M.H., Rooke D.R., Numerical Fracture Mechanics, Computational Mechanics Publications, SOUTHAMPTON, U.K. 1991.

2. Portela A., Aliabadi M.H. & Rooke D.P. “Dual Boundary Element incremental analysis of crack propagation”, Int Journal Computers and Structures, Vol. 46 , pp 237-247, (1993).

3. Mi, Y.; Aliabadi M.H. “Three-dimensional crack growth simulation using BEM”, Computers & Structures, Vol 52, No 5, pp 871-878, (1994).

4. Brebbia C.A., Dominguez J., Boundary Elements - “An Introductory Course”, Computational Mechanics Publications, Southampton, U.K. (1989).

5. Forman, R.G.; Shivakumar, V.; Newman, J.C., “Fatigue Crack Growth Computer Program “NASA/FLAGRO” Version 2.0”, National Aeronautics and Space Administration Lyndon B. Johson Space Center, Houston, Texas, (1993).

6. Paris, P.C., “The Crack Growth of Fatigue Cracks Due to Variation in Load”, PhD. Thesis, Lehigh University, Bethlehen (1962).

7. Erdogan, F and Sih, G.C. “On the crack extension in plates under plane loading and transverse shear”, J. Basic Eng., 85,pp 519-527, (1963).

8. Sih, G.C. “Mechanics of Fracture Initiation and Propagation”, Kluwer Academic, Dordrecht (1991).

9. Flasker J., Pehan S., “Crack propagation in tooth root with variable loading”. Comm in Num Meth in Eng., Vol. 9,pp 103-110 (1993).