

FEM-BEM coupled methodology for cracked stiffened panels

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Abstract

A mixed methodology, involving both BEM and FEM methods, applied on the same domain, is a very effective tool in complex problems, because it is possible to combine the advantages that both of them offer. For example with FEM it is possible to better face non-linear or anisotropic material problems, while BEM is well-suited for crack problems by modeling only the boundaries. In order to create the BEM super-element stiffness matrix for a cracked domain we have adopted a method based on Dual Boundary Element methodology in which it is required to write the dual equation too. When considering a BEM region in a surrounding FEM region, the BEM stiffness matrix is non-symmetric and as such it is appended to the FEM stiffness matrix, in such a way that a direct coupling between a BEM program and MSC/NASTRAN FEM code involves the usage of in house made NASTRAN routines (DMAP). Once the overall stiffness matrix is assembled, the boundary conditions of the BEM super-element are worked out by means of a FEM analysis. For a detailed analysis in the zone surrounding the cracks it is necessary to postprocess the aforesaid boundary conditions with a DBEM procedure so as to obtain the Stress Intensity Factors. A cracked plate example has been analyzed, in order to show convergence aspects, numerical results and related approximation.

1. Introduction

The finite element method (FEM) and boundary element method (BEM) are effective tools in numerical analysis of many physical problems described with a set of partial differential equations and impossible to solve analytically.

In some aspects the two methodologies are complementary, each of them having preferential applications. Namely FEM is well suited to non-linear analysis or anisotropic material while BEM [1] and in particular DBEM (Dual Boundary Element Method) [2, 3, 4] are preferably adopted in fracture mechanics for SIF's (Stress Intensity Factors) evaluation and automatic multiple crack propagation (MSD) [5].

In this context a great research effort is developed aimed to the synergetic coupled usage of the aforesaid methodologies on the same domain, aimed for example to shape optimisation [6-8].

In this work the NASTRAN code is combined with a BEM code [see also 9]: in particular we have used an adapted DBEM code for BEM stiffness matrix evaluation and the BEASY code, in the final part of the algorithm, for cracked plates SIF's assessment.

Actually, if our interest were related solely to the SIF's calculation there would have been no need to use both methodologies because it is possible to do this even with the NASTRAN code alone (although a strong modelling effort would have been necessary), but the perspective is turned to the automatic crack propagation (even if here this problem has not yet been worked out).

With the described subregion procedure, in a crack propagation process it is necessary to iteratively update only the BEM stiffness matrix corresponding to the changeable cracked part of the domain.

2. Theoretical aspects of fracture mechanics and FEM-BEM methods

In fracture mechanics it is possible to outline the following advantages of the BEM with respect to the FEM methodologies: it is sufficient to mesh only the boundary, that makes the modelling process easier, and, by using DBEM, it is possible to solve the cracked domain in a single-region approach and to propagate cracks without repeated time consuming re-meshing. It would be very

cumbersome to mesh, in a FEM procedure, the zone surrounding the crack tips in a cracked domain but the aforesaid procedure is preferable in those parts of the uncracked domain characterized by an inelastic or anisotropic behaviour.

In this context based on the linear elastic hypothesis in the cracked part of the domain there is no account of elastoplastic behaviour of the cracks and related phenomena like crack closure so that the modelling of the region surrounding the cracks through a BEM super-element is straightforward. The remaining part of the domain can be modelled with a FEM super-element and the related stiffness matrix joined to the BEM matrix after a static condensation process of both matrix (or just one) with respect to the interface master nodes. Sometimes the BEM stiffness matrix needs to be symmetrized, before the insertion into the FEM matrix, because it is necessary to exploit the symmetric solver of the FEM codes (in some finite element codes there is no unsymmetric solver available for elastostatic problems) and to minimize run times for solving the algebraic equation system [10]. On the other hand such an approach introduces a non-negligible level of approximation and, above all, is too sensible to the crack mesh used for stiffness matrix evaluation [10]. For this reason, in this work we avoided the artificial symmetrization of the BEM stiffness matrix, using the unsymmetric solver available in NASTRAN code.

3 Stiffness matrix of BEM domain

Our work has been mainly based on a bidimensional DBEM [3] procedure in order to solve a cracked domain in a single region formulation. In this variant of the BEM formulation an independent traction equation is added to the traditional displacement equation in order to avoid ill-conditioning problems in the coefficient matrix of the linear algebraic equations system. This problem arises whenever we try to operate with cracked domain in a traditional BEM single region approach. Moreover discontinuous elements are introduced to model traction discontinuities and to satisfy mathematical conditions related to hypersingular integrals.

The BEM super-element stiffness matrix K is calculated as follows:

- in a BEM problem it is possible to write the following relation between tractions (t) and displacements (u)

$$H * u = G * t ;$$

- since the G matrix is non-singular, it is possible to write

$$t = G^{-1} * H * u ;$$

- nodal tractions need to be converted in nodal forces (f) so as to be consistent with FEM equations and this can be done by premultiplying nodal tractions for a matrix (M) derived from element shape functions

$$f = M * t ;$$

- with this substitution the previous relation becomes:

$$f = M * G^{-1} * H * u ;$$

- furthermore, analogously to the FEM relation $f = K_{FEM} * u$ between nodal displacement and nodal tractions, which are linked by the FEM stiffness matrix K_{FEM} , it is possible to express:

$$K_{BEM} = M * G^{-1} * H$$

- finally the K_{BEM} or the symmetrized K^* matrix for the BEM domain will be assembled with the stiffness FEM matrix.

4. Unsymmetric procedure and problem definition

The following approach has been followed for the SIF's calculations in a substructured cracked plate (Fig.2), applied to a particular configuration of the BEM domain but of general validity:

- the cracked domain region is treated as a BEM super-element of which it is necessary to calculate the related stiffness matrix;
- the remaining part of the plate is modelled with finite elements (Fig.3), assuming a sufficiently large dimension of the plate which is approximated as infinite in the theoretical problems examined in this work;
- the BEM super-element stiffness matrix, after condensation, has been inserted into the FEM stiffness matrix, exploiting an in house made DMAP routine for NASTRAN code;
- by running a NASTRAN analysis it has been possible to calculate the BEM-FEM domain interface nodal displacements: actually another DMAP was necessary in order to force NASTRAN using the routine SOLVE instead of the normal routine for system matrix solution (the latter was affected by a bug when used with a non-symmetric stiffness matrix);
- the interface nodal displacements constitute the input boundary conditions for the super-element BEM cracked domain (Fig. 2), whose SIF's will be calculated by means of a DBEM code (BEASY).

One of the main drawbacks of the procedure based on an artificial symmetrization of the Boundary Element (BE) sub-domain stiffness matrix, is the high sensitivity of results to the crack mesh used when calculating the BE stiffness matrix. This is due to the lack of mathematical foundation in the procedure of BE stiffness matrix symmetrization, in such a way that the aforementioned approximate procedure is recommended just for a first insight into the problem of SIF's evaluation and when the (Finite Element) FE code is not capable of processing an overall un-symmetric stiffness matrix [8].

NASTRAN code provide an unsymmetric solver for linear system of equations, so that a routine ("DMAP") has been developed, capable of receiving the unsymmetric BEM stiffness matrix and to process it together with the FEM stiffness matrix.

The correctness of the procedure has been assessed with reference to the SIF's evaluation on the following problem:

- a multi-site damaged plate, whose dimensions are width $W=3400$ mm and height $H=3400$ mm, has been chosen (Fig. 2), with the central cracked part modelled with a BE sub-domain (Fig. 4) and the remaining part (Fig.3) with the Finite Element method (FEM);
- the panel is made of bare 2024 T3 aluminium alloy with Young modulus $E=72000$ MPa and Poisson ratio $\nu = 0.3$, thickness $t=1.27$ mm; it is subject to a uniform remote tension stress of magnitude 100 MPa (pure mode I load);
- results are based on Linear Elastic Fracture Mechanics assumptions;
- the cracked region (Fig.4), whose dimensions are width $W=340$ mm and height $H=400$ mm, is treated as a BEM super-element of which it is necessary to calculate the stiffness matrix, without the need to force the symmetry of this matrix, because NASTRAN code provide an option that allows a static analysis with an overall un-symmetric stiffness matrix (even if the solver is clearly less efficient).

5. Numerical results and comparisons

The results obtained with the aforementioned "un-symmetric" FEM-BEM procedure have been compared with those obtained from a Boundary Element Code (BEASY) in a full BEM analysis as reported in Table 1. The crack mesh adopted (that is the same for FEM-BEM and full BEM analysis) for the final SIF's assessment is based on 2 equal elements per crack side for the short cracks and 4 graded elements for the main crack as shown in Fig.3.

Crack tip abscissa X (mm)	K_I (MPa mm ^{1/2}) FEM-BEM	K_I (MPa mm ^{1/2}) BEM
60	1433	1433
103	778.3	778.4
128	743.8	743.0

Table 1

6. Conclusions

In conclusion, it is important to remark the lack of mathematical foundation for the BEM stiffness matrix artificial symmetrization, which the approximate FEM-BEM “symmetric” procedure relies on. This approach is just recommended when an un-symmetric solver is not available in the FEM environment.

On the contrary the latter procedure (no symmetrization) is of general validity as it appears from the presented example and only apparently cumbersome because it can be easily integrated, reducing significantly manual intervention, so as to gain industrial applicability. Further investigations are necessary with reference to more complex problems (e.g. cracked stiffened plates, multiple crack propagation, etc.).

In particular such a procedure could result very effective for crack propagation problems, where the damaged part is limited to restricted areas of the domain: in fact, with reference to the non-changeable FEM domain, it is possible to adopt a condensing / non-condensing technique to partition the unknowns into condensed and master unknowns. An exact equation can be written which relates the condensed unknowns to the master unknowns, thereby eliminating the condensed from the FEM system matrix during the crack propagation in the BEM sub-domain. However, an expansion process at the end is required to retrieve the values of the condensed unknowns.

Another important advantage of the condensing / non-condensing technique is related to iterative processes where each iteration can be performed over the condensed system matrix leaving the retrieval of the condensed unknowns until the final iteration, with obvious reduction in computer resources.

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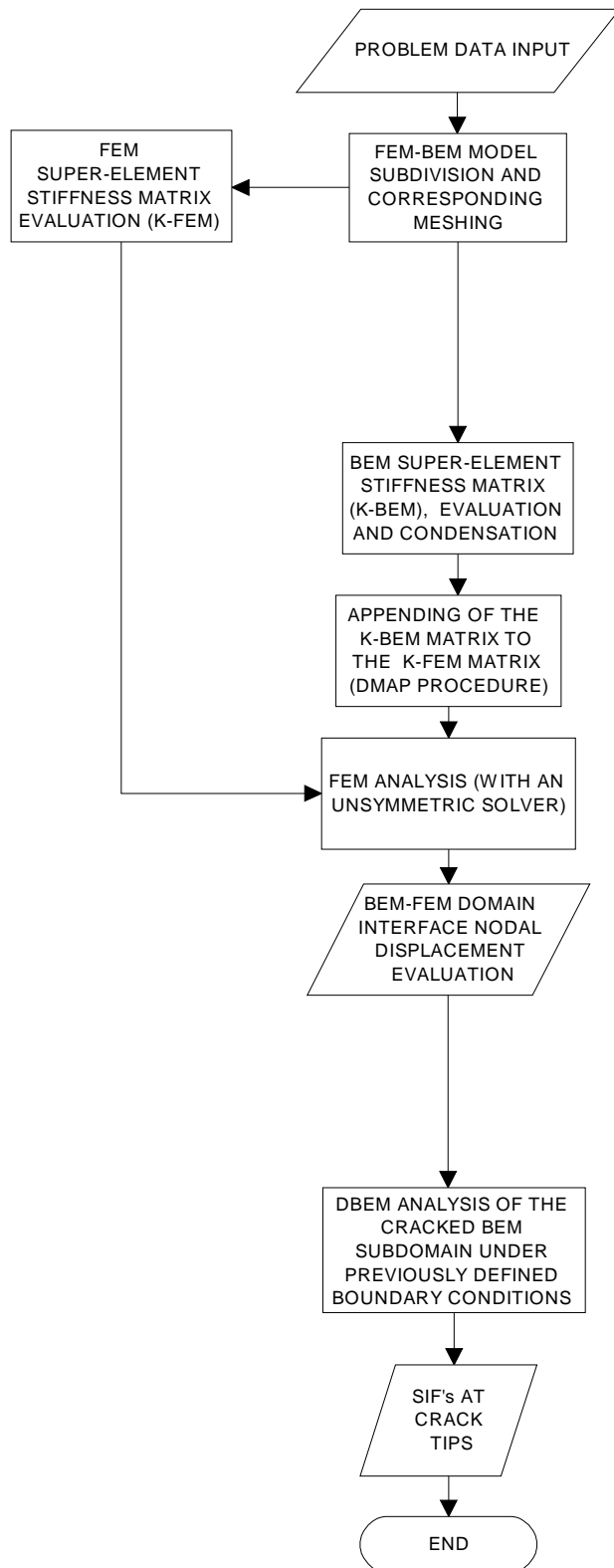


Fig. 1: Flow chart of the procedure.

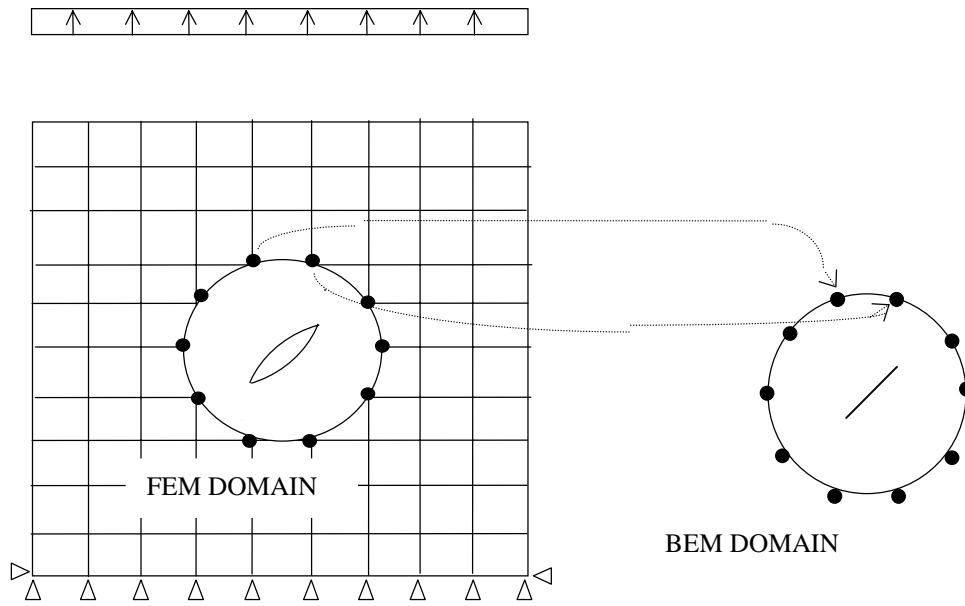


Fig.2: Multi-site damaged panel, loaded with a traction $t=100$ MPa.

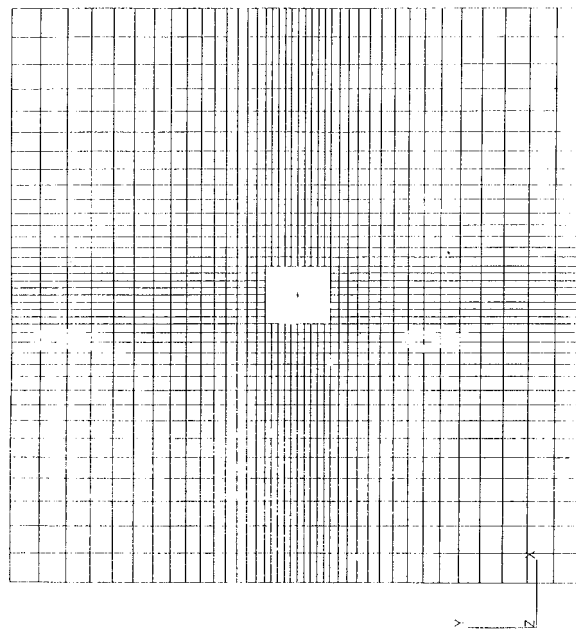


Fig. 3: FEM domain and related mesh.

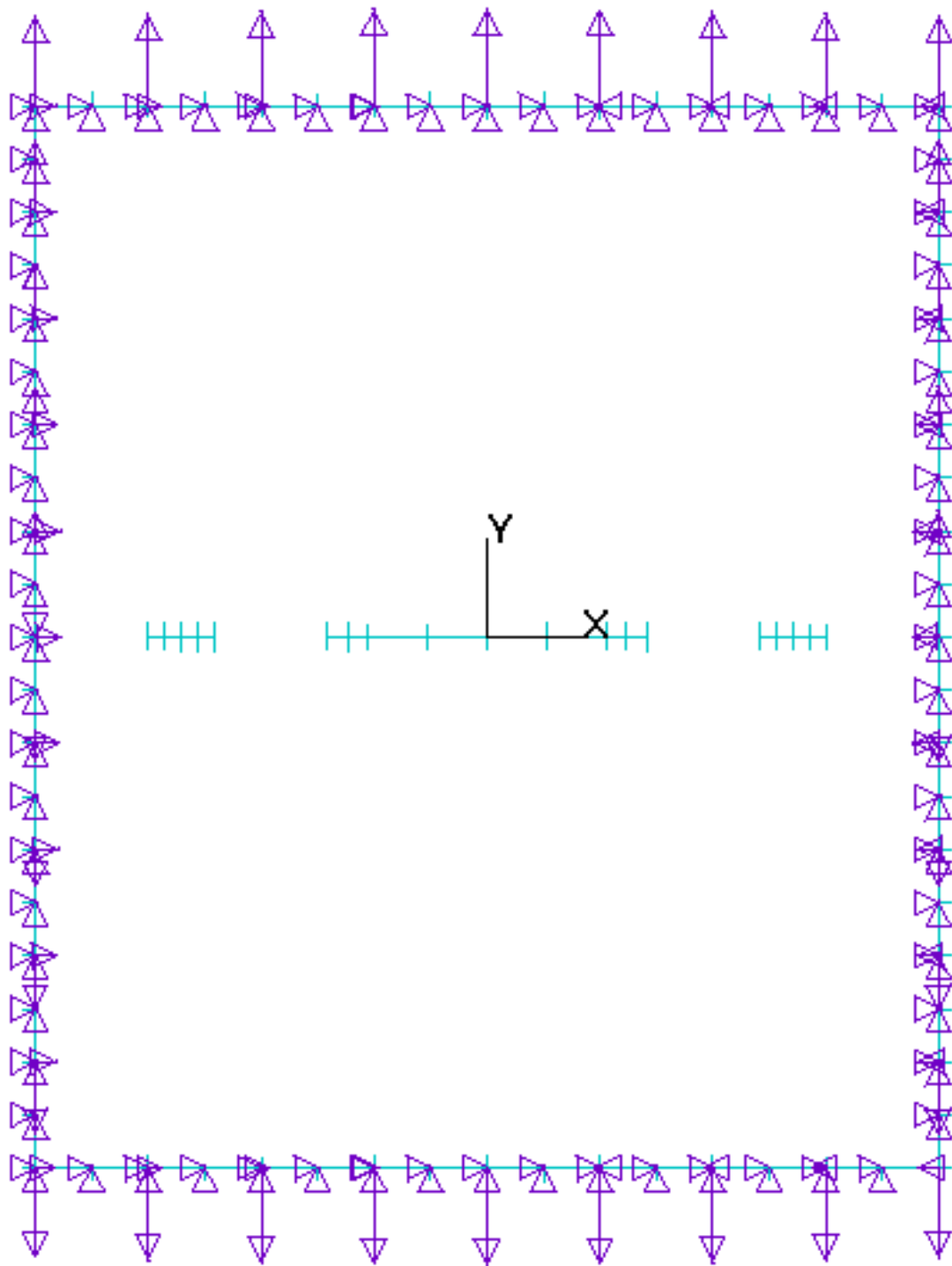


Fig. 4: Close-up of the BE cracked sub-domain and related displacement boundary conditions.