ANALYSIS OF FLUID STRUCTURE INTERACTION OF SPACECRAFT'S FLUID TANKS USING COMBINED BOUNDARY ELEMENT AND FINITE ELEMENT METHODS

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ABSTRACT

The fluid tanks play an important role in the dynamic behaviour of spacecraft as they carry fuels which may take up to 50% of the total spacecraft mass.

The fluid tanks are therefore needed to be adequately modeled in order to obtain an accurate prediction of spacecraft dynamic behaviour.

This paper presents an algorithm by which a Boundary Element and a Finite Element systems are combined to solve the problems of fluid - structure interaction of fluid tanks. The fluid is assumed to be incompressible and motion limited to small amplitudes.

It is shown that a combined system can provide an optimum analysis environment in which Finite Element method is used to model the thin walled tank and Boundary Element method to represent the fluid in the tank.

Based on this idea, the authors developed a coupled system which employs BEASY, Boundary Element and the ASKA, Finite Element analysis modules to generate the fluid and structural matrices. These matrices are then combined along the wetted surface between the tank and fluid by considering the appropriate equilibrium and compatibility conditions.

Example problems presented include analyses of a full fluid filled tank, a cylindrical canal and a typical spacecraft's fluid tank.

1. INTRODUCTION

In the last two decades, the Finite Element method (FEM) has established itself as a powerful analysis tool in many areas of engineering analysis including that of fluid-structure interaction problems. The main feature and reason for the popularity of the (FEM) has been its wide range applications.

The main drawbacks of using FEM has always been its requirement of modeling the domain of the problem, which results in a significant number of internal degrees of freedom at which the solution is computed. In practice, unless one is particularly interested to know the solution through the entire domain, the introduction of internal degrees of freedom only contributes to a wasted modeling and computational effort. Overcoming these problems has been the motivation of many Researchers to discover an alternative analysis technique (Ref 2) like Boundary Element Method. Based on boundary integral equations (for which its theory was known well before the introduction of first multipurpose commercial Boundary Element analysis system BEASY in 1981) it only requires a model of the boundary and introduces internal degrees of freedom only in the area of interests. This feature has attracted extensive research into the technique in the past ten years, extending its application to most engineering problems. In practice, however, it has its own drawbacks when dealing with non-linear problems and thin structures.

2. WHY USE A COMBINED SYSTEM FOR FLUID STRUCTURE INTERACTION PROBLEM?

In general the fluid structure interaction problems are divided into two categories:

a. <u>External problems</u> for which the volume of the fluid is infinite, such as those found in offshore and marine structures (Figure 1).

b. <u>Internal problems</u> for which the boundary of the fluid is bounded like the problems of fluid tanks (Figure 2).

It is important to note that in both cases the boundary of the fluid coincide with that of the structure. One can therefore draw an important conclusion that representing the fluid using Boundary Element method does not require any extra modeling effort than defining the structure. The structural geometric data at structure-fluid interface can be directly used to model the fluid. For structure however it is best to use Finite Element method as these problems are normally involved with thin shells for which Boundary Element technique may not be cost effective.

The use of combined technique is particularly important for two classes of fluid structure interaction problems. First those for which the volume of the fluid is significant compared with its total surface area, like external problems. Second for which the fluid is subject to some internal geometric complexities, such as problems of fluid tanks with internal baffles.

The fluid tanks of spacecraft fall into both categories as the size of these tanks are normally large and also can be complex due to existence of baffles (Figure 3).

3. MODELING ASPECTS

Apart from the pre and post processor used, a combined system would involve four modules as showing in (Figure 4) . The interface programs shown can be integrated into one module, however they are referred to as two programs for the sake of simplicity. The role of these programs are particularly important as we will discuss later.

The analysis flow is as follows:

3.1 Initial FE complete model.

Initially the complete model of structure and fluid is prepared within Finite Element environment. In this model both the fluid and the structural elements are defined within the same net. The structural and fluid (wetted or free surface) elements are defined by means of the assignment of different groups.

In the initial complete FE model, the fluid elements are defined by regenerating the structural shell elements at the interface between the structure and the fluid. During this process the fluid elements are converted to membrane elements. However the fluid elements on the free surface are modeled independently.

3.2 Fluid Model

The next step is to extract the Boundary Element fluid model from the initial Finite Element complete model.

Unfortunately this cannot be done directly due to differences in modeling concept between Finite Element and Boundary Element methods. The role of the FE-BE interface program is therefore to preserve the compatibility between the FE and BE fluid models. This would require modification to the FE fluids data in the initial complete model. (Figure 5)

These are:

* Modification to Element Connectivities:

As the Boundary Element method deals with the boundary of the problem, it requires a means by which it can distinguish an external boundary from an internal boundary, i.e. the element should be defined such that it represents the right direction of the outward normal. The direction of Element connectivity provides this means and FE-BE interface program should reverse this if necessary. (See Figure 6)

* Introducing Additional FE Nodes:

While the structure and the fluid elements are coupled along the wetted surface, the free surface should be allowed to move freely. This decoupling is enforced by introducing double nodes at interface between the free and the wetted surfaces of the fluid (Figure 7) .

* Defining appropriate Fluid Elements Continuity:

In general, a numerical modeling technique involves two approximations:

a) Geometric Approximation:

This is made by subdividing the domain (or the boundary as in the case of Boundary Element method) into number of elements, within which the geometry is approximated in terms of geometric parameters of some predefine points on the element's boundary called mesh points.

b) Solution Approximation:

This involves approximating the problems variables within each elements in terms of their values at some predefined point called nodal points.

The number of mesh points and nodal points are chosen depending on the required order of geometric and solution approximation.

In Finite Element the mesh and nodal points are chosen to be the same and because of the inter-element continuity requirement, elements are defined such that they are matched at mesh points along their common boundary.

The Boundary Element method however relaxes this requirement mesh and nodal points do not have to coincide and mesh grading does not have to be continuous across the boundary. (Figure 8)

The continuous or discontinuous elements can be chosen arbitrarily however, an optimum model is based on continuous elements with discontinuous elements used in the areas where natural discontinuities of problem variables are expected.

In this application the normal pressure is discontinuous along any geometric kinks and at interface between the wetted and free surfaces. The task of FE-BE interface program is therefore to consider discontinuity where appropriate.

The details of all modifications to the initial FE fluid model is saved and used later tc create the final FE model. (Figure 9)

3.3 The Fluid Mass and Stiffness Matrices:

The modified fluid model is now passed to the BE analysis model. The mass and stiffness matrices are generated initially at BE nodes and then extrapolated to the FE mesh points and symmetrized due to unsymmetrical nature of Boundary Element influence matrices (Ref 1) (Figure 10).

3.4 Final FE Complete model:

The final FE model is prepared by the EE-FE interface program. The FE structural model from the initial complete model together with the modified FE fluid models of each tank are passed to the final model. The model fluid of each tank and structural model are now defined within different nets and tangential degrees of freedom on the free surfaces are suppressed. (Figure 11)

3.5 Final Analysis:

This is carried out within FE environment. The final complete model is passed to FE analysis module, for which the mass and stiffness of the structure is calculated and combined with those of fluid, and used to evaluate the dynamic properties. (Figure 12)
Note that the modeling tasks of FE-BE and BE-FE interface programs are carried out automatically and are transparent to the user.

4. THEORETICAL ASPECTS

Early studies on coupled BEM-FEM techniques were carried out by Zienkiewicz (Ref 3) and Brebbia (Ref 4). The technique was later used by Walker (Ref 5) to solve the fluid-structure interaction problems of the type mentioned in this paper. However the early work on this subject was based on BEM flat constant elements. The formulation used also took into account the fluid stiffness contribution from the free surface only nd therefore could not deal with problems of fluid filled tank. The formulation used in this study is based on:

4.1 Boundary Element Formulation:

The potential field within an incompressible fluid is governed by the Poisson equation,

$$\nabla^2 u = p \qquad (1)$$

u is the velocity potential

p is the source density

The velocity vector can be expressed as the gradient of the velocity potential.

$$\underline{q} = \nabla \underline{u} \qquad (2)$$

Assuming that the velocity potential u is continuously differentiable twice in Ω and Γ (where Ω and Γ correspond to any positions within the domain and on the boundary respectively) then from divergence theory one can write

$$\int \nabla \underline{u}.\underline{n} d\Gamma = \int \nabla^2 \underline{u} d\Omega \qquad (3)$$

Let us now consider the two arbitrary continuously differentiable potential fields u and u^*.

Applying the divergence theorem to the product of $u^* \nabla^2 \underline{u}$ and $u \nabla^2 \underline{u}^*$ gives

$$\int u^* \nabla^2 \underline{u} d\Omega = \int u^* \nabla \underline{u}.\underline{n} d\Gamma = \int u^*.\nabla \underline{u} d\Gamma \qquad (4)$$

$$\int u \nabla^2 \underline{u}^* d\Omega = \int u \nabla \underline{u}^*.\underline{n} d\Gamma = \int u.\nabla \underline{u}^* d\Gamma \qquad (5)$$

subtracting (5) from (4) and considering (1) gives

$$\int u \nabla^2 \underline{u}^* d\Omega = \int (u \nabla \underline{u}^*.\underline{n} - u^* \nabla \underline{u}.\underline{n}) d\Gamma - \int u^* \rho d\Omega \qquad (6)$$

Assume that u is defined as the potential field due to a unit source at a point x_i ($x \in \Omega$) in the domain. The unit source can be presented by a distributed source whose density is given by a Direc Delta function of $\Delta(x - x_i) = \Delta_i$.

This function has the following properties

$$\Delta(x - x_i) = 0 \quad \text{for} \quad x \neq x_i$$

$$\int \Delta(x - x_i) d\Omega = \int_{x_i - \epsilon}^{x_i + \epsilon} \Delta(x - x_i) d\Omega = 1 \qquad (7)$$

$$\int f(x) \Delta_i d\Omega = f(x_i)$$

U^* can therefore be defined as the solution of the following equation and is called a Fundamental Solution

$$\nabla^2 u^* = \Delta_i \qquad (8)$$

Substituting (8) and (6) gives

$$\int u \Delta_i d\Omega = \int (u \nabla \underline{u}^*.\underline{n} - u^* \nabla \underline{u}.\underline{n}) d\Gamma - \int u^* \rho d\Omega \qquad (9)$$

Substituting the expression for the velocity (2) and (7.3) into (9) and in the absence of any domain source one can arrive at

$$c_i u_i = \int u q^* d\Gamma - \int u^* q d\Gamma \qquad (10)$$

where u is the velocity potential, q is the normal velocity, u^* is the fundamental solution, and q^* is the normal derivative of the fundamental solution.
The fundamental solution for 3D problems can be found to be

$$u^* = \frac{1}{4\pi R}$$

u^* is the potential field at point due to application of a unit source at point i. R is the distance between x and x_i.

By taking into the boundary and subdividing the boundary into a number of elements within which the potential field u is approximated at element nodal points then one can write equation (10) in the following matrix form

$$\underline{H}\,\underline{U} = \underline{G}\,\underline{Q} \qquad (11)$$

where $\underline{H}$ and $\underline{G}$ are boundary integral influence matrices. $\underline{U}$ and Q are boundary nodal values of velocity potential and normal velocity.

4.2 Derivation of fluid's Mass and Stiffness Matrices

Assuming that the fluid's dynamic movement is harmonic one can express the normal displacement by

$$W(t) = e^{i\omega t} \underline{W} \qquad (12)$$

and therefore the normal velocity can be written by

$$\underline{Q} = \frac{\delta W(t)}{\delta t} = i\omega e^{i\omega t} \underline{W} = i\omega \underline{W}(t) \qquad (13)$$

Also from the Bernoullits equation for harmonic motion the following relationship can be used between the pressure and the velocity potential

$$\underline{U} = -\frac{1}{i\omega\rho} \underline{P} \qquad (14)$$

Substituting the velocity potential from (13) ir.to (11)

$$\underline{H}\,\underline{U} = i\omega \underline{G}\,\underline{W}(t) \qquad (15)$$

Substituting the velocity potential from (14) into (13) one can obtain the following relationship between the pressure and the displacements.

$$\underline{H}\,\underline{P} = \rho\omega^2 \underline{G}\,\underline{W} \qquad (16)$$

Normal nodal forces can be expressed in terms of pressure by

$$\underline{F} = \underline{N}\,\underline{P} \tag{17}$$

where $\underline{N}$ is distribution matrix and $\underline{F}$ is the nodal normal force vector.

Substituting (15) into (14) one can write

$$\underline{F} = \omega^2 \underbrace{[\rho \underline{N}\,\underline{H}^{-1}\underline{G}]}_{M}\underline{W} = \omega^2 \underline{M}\,\underline{W} \tag{18}$$

where M is the equivalent mass matrix which is unsymmetric due to the **unsymmetric** nature of the Boundary Element influence matrices of H and G. Note that $\underline{F}$ and $\underline{W}$ are the nodal inertial forces and **displacement** in nodal normal co-ordinates. The expression for the stiffness can be obtained by considering the contribution of gravity term in the Bernoulli's equations to normal pressure

$$\underline{P} = -\rho g \underline{W}_z$$

where W_z is displacement in the direction of gravity
Transforming pressure to normal forces gives

$$\underline{F} = -\rho g \underline{N}\,\underline{W}_z \tag{19}$$

From (18) and (19) the equilibrium equation can be written by

$$\underline{F} = \omega^2 \underline{M}\,W - \underline{K}\,\underline{W}_z \tag{20}$$

where W_z in (19) is expanded to represent displacement in Global co-ordinate system
Transforming $\underline{F}$ and $\underline{W}$ from the local to the global co-ordinate system

$$\underline{F}_G = \underline{T}^T \underline{F} \quad \text{and} \quad \underline{W} = \underline{T}\,\underline{W}_G \tag{21}$$

substituting (21) into (20)

$$\underline{F}_G = \omega^2 \underline{T}^T \underline{M}\,\underline{T}\,\underline{W}_G - \underline{T}^T \underline{K}\,W_G \tag{22}$$

or

$$\underline{F}_G = (\omega^2 \underline{M}_G - \underline{K}_G)\,W_G \tag{23}$$

5. APPLICATION

In this work, a coupled system was developed, based on Boundary Element multi-purpose program BEASY and Finite Element program ASKA. (Flowchart 1).

The analysis of the problems presented in this paper are carried out by modeling the problem initially within ASKA environment.

The initial ASKA model is then passed to BEASY-GIS program to create the modified ASKA and BEASY fluid models.

The fluids mass and stiffness matrices are evaluated by BEASY and passed to ASKA together with a modified ASKA fluid model, where the final dynamic analysis is carried out.

For the problem of rigid tanks the full analysis can be carried out with BEASY Boundary Element environment (Flow Chart 2).

5.1 Analysis of a full fluid filled flexible spherical tank

In this example, the analysis of a full spherical tank of diameter 0.2m filled with the liquid of 1000 Kg/m3 density is investigated (Figure 13).
The tank itself is assumed to be of 1mm thickness with young's nodules, poisson ratio and mass density of 7.0E+10 N/m 0.3 and 2800 Kg/m3 respectively.

The acceleration of gravity is assumed to be 9.81 m/sec2 and the tank is assumed to be clamped at 6 d.o.f along a circumferential circle.

For this analysis 48 linear triangular and 64 linear quadrilateral shell elements are used to model the tank and identical membrane elements are used to model the fluid.

For this problem there is no sloshing modes as the tank is full with no free surface. The modes are purely hydrostatic which are shown in (Figure 14).

The same model is solved for the case when the tank is partially filled up to 0.05 (Figure 15). The results obtained agree with those given by Steelant (Ref 6) with errors between 2% to 4% for various mode. More accurate results could be obtained using curved elements. The sloshing nodes are shown in (Figure 15).

5.2 Analysis of Cylindrical Canal

In this application, the sloshing behaviour of the liquid of 1000 Kg/m3 density in a half full cylindrical canal of 15 metre long and 5.4 metre diameter is studied (Figure 16).

The canal's wall is assumed to be 5mm thick with young's modulus, poisson ratios and mass density of 2.0E+10 N/m, 0.3 and 7800 Kg/m3 respectively.

The acceleration of gravity is assumed to be 9.81 m/see.

The total number of 108 linear quadrilateral shell and 78 linear quadrilater of membrane elements were used to model the canal and the liquid respectively.

The sloshing mode obtained is shown in (Figure 17).

5.3 Analysis of a typical spacecraft's tank

In this example the analysis of large a fluid tank which is typically used as part of the spacecraft is investigated.

The tank is assumed to be 16.7m high, 5.0mm thick with diameter of 5.4m and made of aluminium alloy with young modules, poisson ration and mass density of 71000 N/mm2, 0.3 and 3253 Kg/m3 respectively.

The tank is filled with liquid hydrogen of 70 Kg/m3 mass density.

For this element 224 linear quadrilateral shell and membrane elements were used to model the structure and wetted surface of the fluid, and 32 linear quadrilateral membrane elements to model the free surface (Figure 18).

The first 6 modes are shown in Figures (19), (20) and (21). The first two modes correspond to lateral fluid modes i.e. sloshing modes. The next two modes are the axial saddle modes of the fluid. The 5th and 6th modes are the first interaction modes:

6. CONCLUSIONS

The combined Finite Element - Boundary Element method provides the most cost effective environment for the analysis of fluid structure interaction problems including that of fluid tanks.

Using this technique, the structure is modeled using Finite Elements and the fluid is presented by Boundary Elements. This technique would introduce no additional modeling effort nor any extra degrees of freedom to that of the structures (with the exception of the free surface) and therefore minimizes the man and computer resources required.

Acknowledgements:

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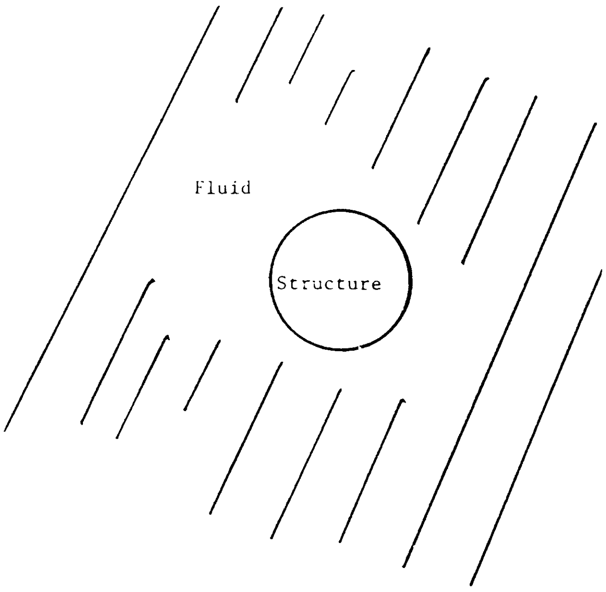


Fig. 1 Typical model of an external problem

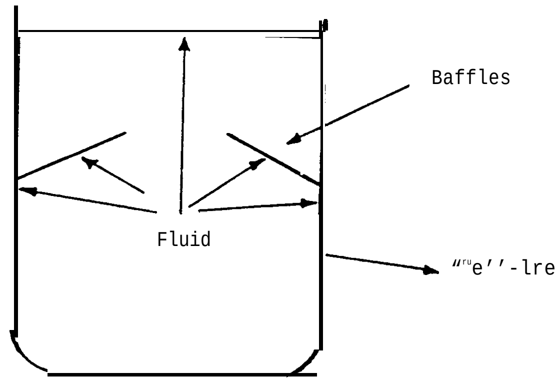


Fig. 3 Model of a fluid tank with baffles

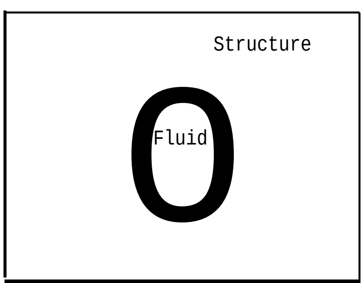


Fig. 2 Typical model of an internal problem

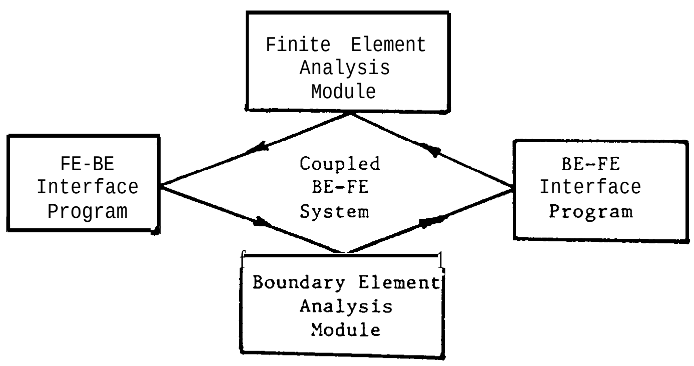


Fig. 4

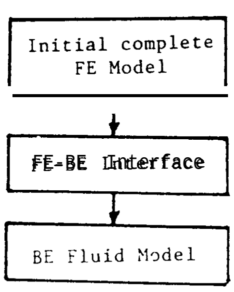


Fig. 5

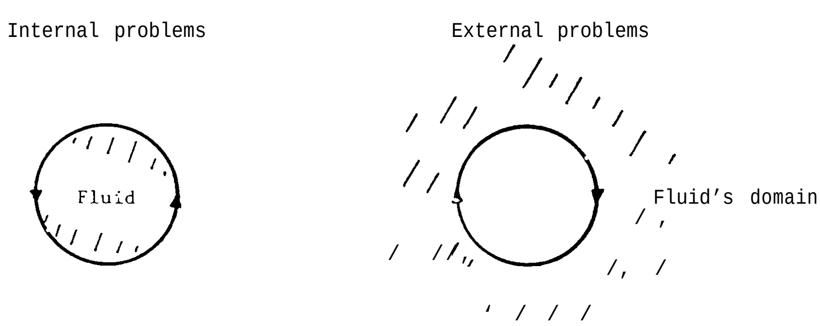


Fig. 6 The conventional direction of connectivities for internal and external problems

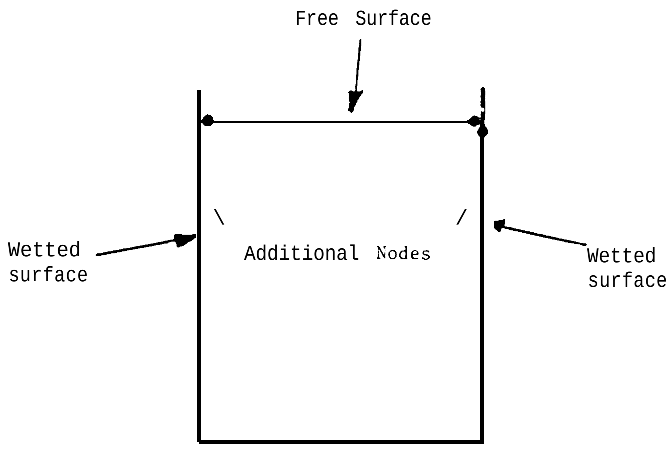


Fig. 7 Boundary element model of a fluid tank with double mesh points along the interface between the wetted and free surface

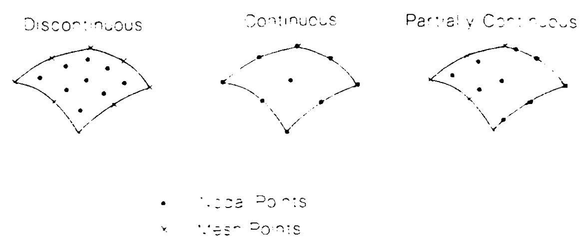


Fig 8a Boundary Elements

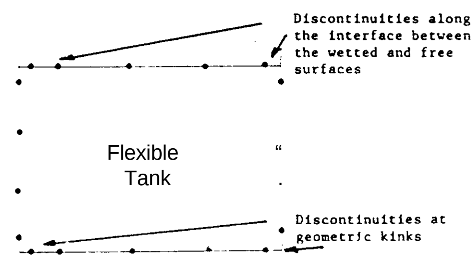


Fig. 8b Boundary Element model of a flexible tank

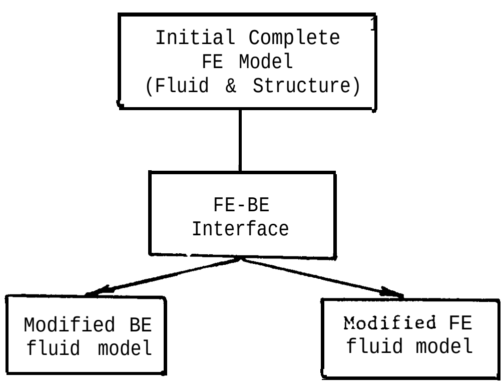


Fig. 9

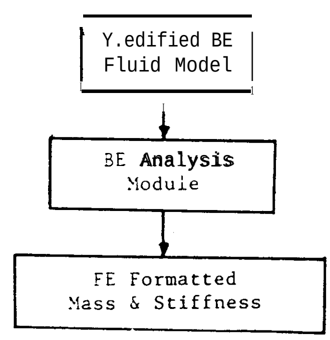


Fig. 10

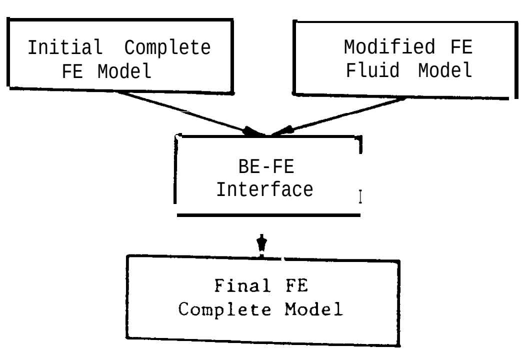


Fig. 11

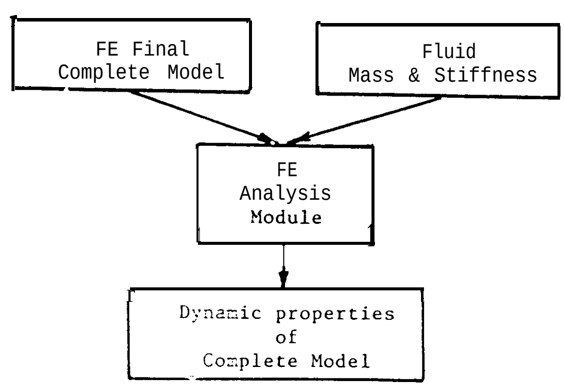


Fig. 12

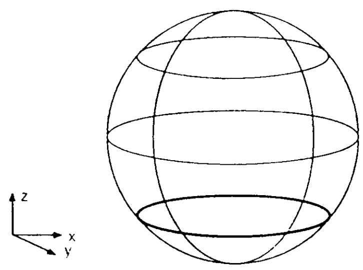


Fig. 13 Geometry of the Full Fluid filled spherical tank

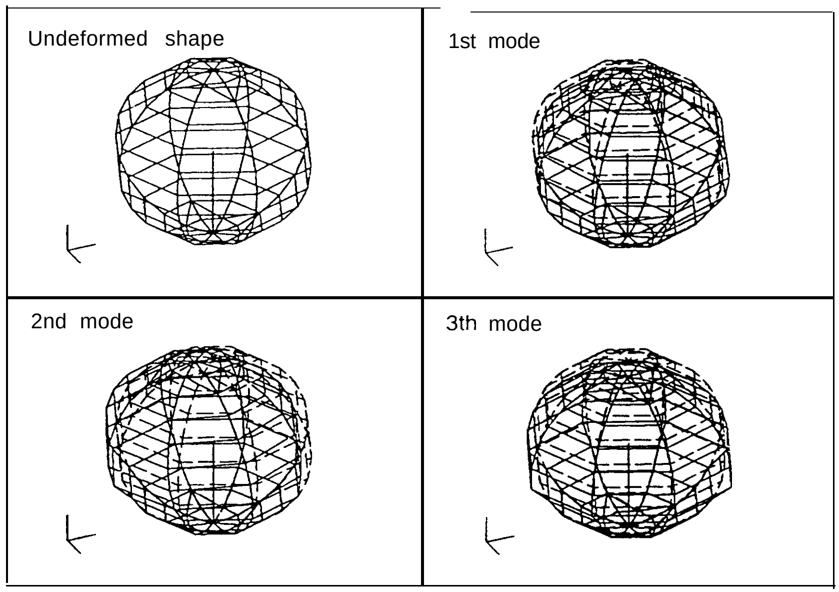


Fig. 14 Mode of vibrations of Full fluid filled spherical tank

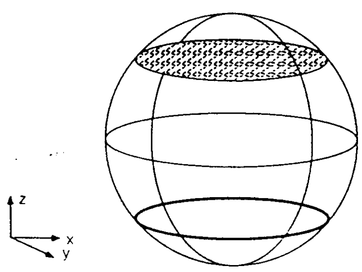


Fig. 15a Geometry of the partially filled spherical tank

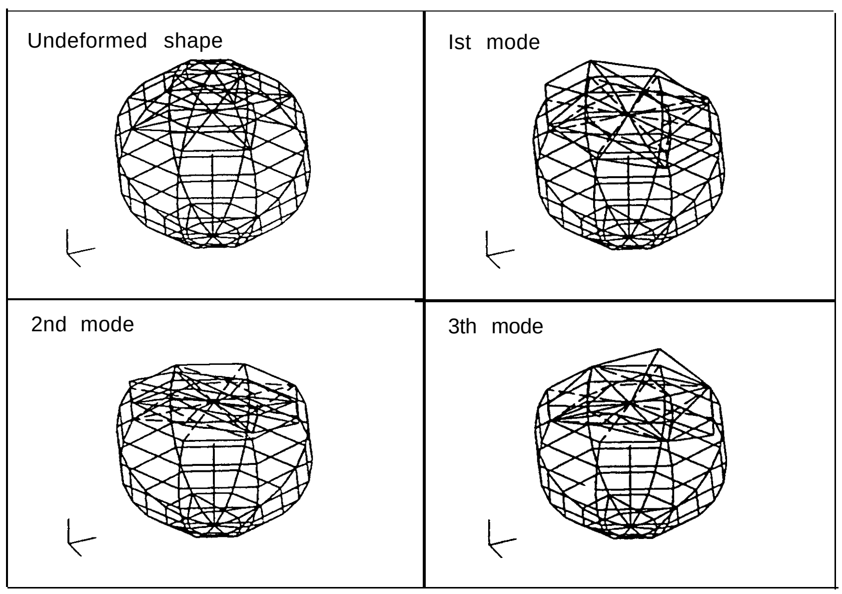


Fig. 15b Modes of vibrations of partially filled spherical tank

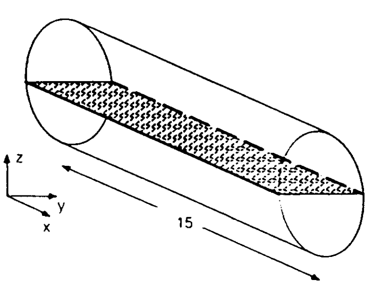


Fig. 16 Geometry of a Cylindrical canal

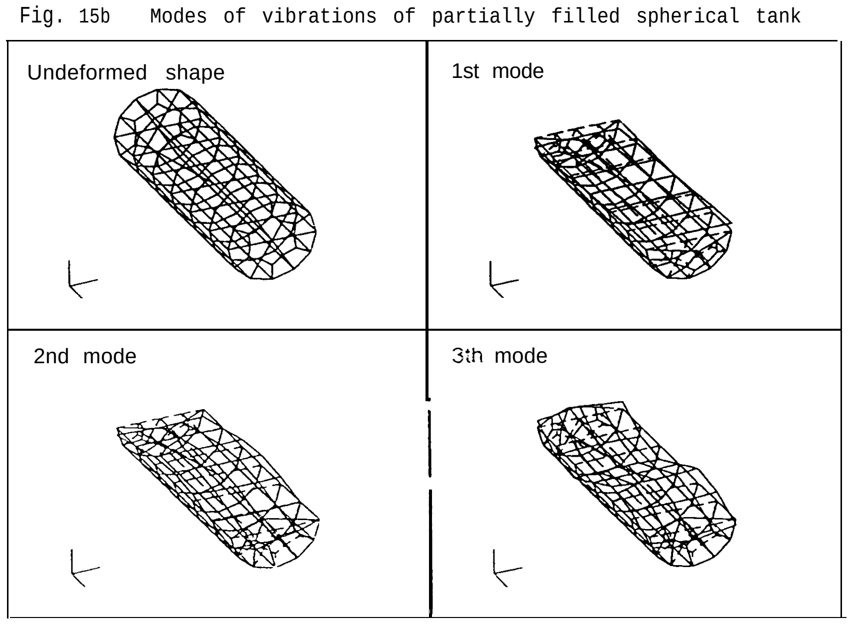


Fig. 17 Mode of vibrations of the Cylindrical canal

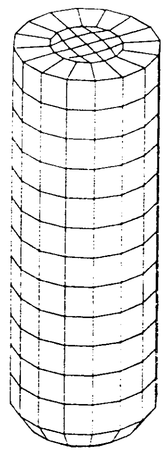


Fig. 18 Boundary Element model of a typical spacecraft large tanks

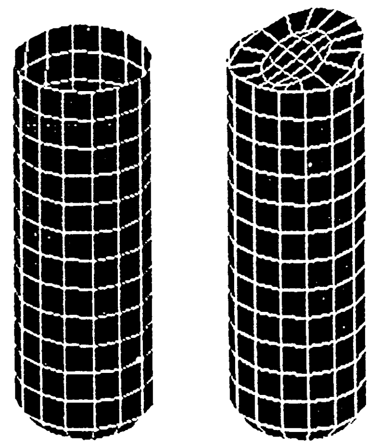


Fig. 19 Lateral sloshing fluid modes of the large tank

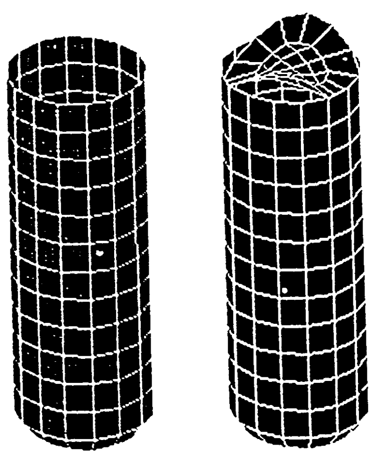


Fig. 20 Fluid's saddle mode of the large tank

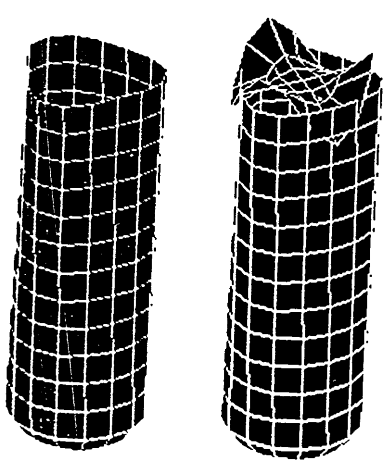
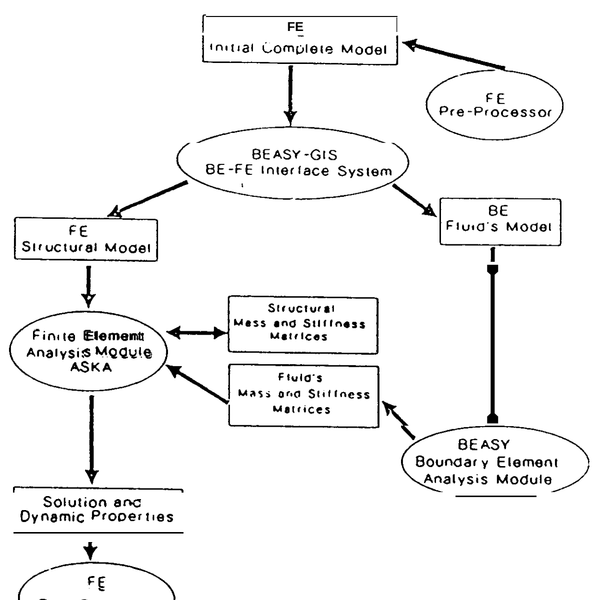
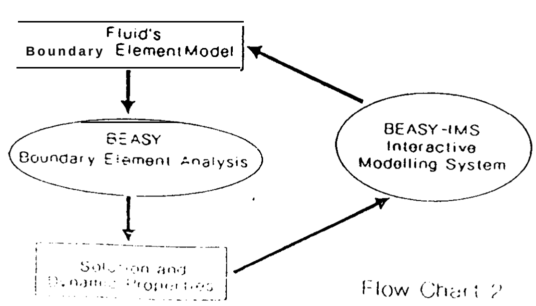


Fig. 21 The first structure interaction mode of the large tank

(Analysis of Problems with Flexible Containers)



Flow Chart 1



Flow Chart 2