Predicting Residual Strength Using Fully Automatic Crack Growth

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Abstract

In recent years the need to predict how a product will perform over the life cycles has increased. Aerospace and other industries need to demonstrate the durability of designs and plan effective maintenance schedules. The growth of cracks due to fatigue loads plays an important role in limiting the life of products. In this paper, a new method is presented for the automatic growth of edge cracks in three-dimensional fracture analysis using BEM. The procedure described overcomes the current problems with crack growth analysis that currently has to be performed manually. Applications are presented which demonstrate the effectiveness of the technique.

Introduction

The cost to industry of fracture was recently estimated in a report of the US Department of Commerce entitled "The economic effect of fracture in the United States". In this report it estimated that the cost of fracture was $119 billion dollars per year (4% of gross national product). It further estimated that approximately one third could be saved through the use of current fracture control technology and a further 25% could be saved through fracture related research. Therefore the annual cost of fracture could be halved by the application of better design tools based on fracture mechanics technology.

The analysis of cracks within structures is an important application if the damage tolerance and life of structures and components are to be predicted. Often cracks cannot be avoided in structures, however the fatigue life of the structure depends on the location and size of these cracks. In order to predict the fatigue life for any component, a crack growth study needs to be performed.

Durability and damage tolerance calculations are typically based on experimental data or analytical studies of simplified cases. In this paper, a new technology is described based on the boundary element method that can be used not only to compute the stress intensity factors for complex three dimensional components and structures, but also automatically predict how cracks will propagate.

The benefits of the approach are numerous, accurate prediction of crack growth, improved stress intensity data and more accurate prediction of life, thus providing a tool which can be used to investigate damage control strategies and optimise durability.

The boundary element method is an ideal solution for performing crack growth analysis due to the high accuracy of the stress results computed on the surface of the structure [1-7]. In addition, since only the boundary of the body needs to be discretised for boundary element analysis, the meshing time can be significantly reduced over other analysis methods.

For a few years now, the BEASY analysis code has been capable of performing fully automatic crack growth of embedded cracks[1,6]. For an embedded crack the addition of new elements as the crack grows can be performed automatically as the new elements are not connected to the surface mesh. This is another benefit of the boundary element method.

In the case of an edge crack, however, only a single crack growth step could be performed. The user was then required to generate a new crack analysis model from the predicted crack front and repeat the process for each required step. This can often be a complicated task and requires a significant amount of the users time.

In this paper, a new method is presented for growing edge cracks fully automatically. In addition, the method allows the user to initiate a crack into a model from a library of crack shapes, removing from the user the task of meshing the crack.

Fracture Analysis

Catastrophic fracture failure of engineering structures is caused by cracks that extend beyond a safe size. Cracks, present to some extent in all structures, either as a result of manufacturing fabrication defects or localised damage in service, may grow by mechanisms such as fatigue, stress-corrosion or creep. The crack growth leads to a decrease in the structural strength. Thus, when the service loading cannot be sustained by the current residual strength, fracture occurs leading to the failure of the structure. Fracture, the final catastrophic event that takes place very rapidly, is preceded by crack growth which develops slowly during normal service conditions, mainly by fatigue due to cyclic loading.

Damage tolerance assessment is a procedure that defines whether a crack can be sustained safely during the projected service life of the structure. Damage tolerance assessment is, therefore, required as a basis for any fracture control plan, generating the following information, upon which fracture control decisions can be made:

- **The effect of cracks on the structural residual strength, leading to the evaluation of their maximum permissible size.**

- **The cracks growth as a function of time, leading to the evaluation of the life of the cracks to reach their maximum permissible size, from which the safe operational life of the structure is defined.**

Linear elastic fracture mechanics [7] can be used in damage tolerance analyses to describe the behaviour of cracks. The fundamental assumption of linear elastic fracture mechanics is that the crack behaviour is determined solely by the values of the stress intensity factors which are a function of the applied load and the geometry of the cracked structure. The stress intensity factors, thus play a fundamental role in linear elastic fracture mechanics applications.

Crack-growth processes are simulated with an incremental crack-extension analysis. For each increment of the crack extension, a stress analysis is carried out and the stress intensity factors are evaluated. The crack path, predicted on an incremental basis, is computed by a criterion defined in terms of the stress intensity factors.

Theoretical Foundation

A number of authors have studies the numerical simulation of crack growth using a variety of numerical techniques. Finite element methods have been developed[10,11,12] using mesh generation techniques and cohesive elements. They however still require a discretization of the three-dimensional volume mesh. Boundary element solutions[1-10] benefit from a surface only representation of the crack.

The method proposed uses the Dual Boundary Element Method (DBEM) to predict the stress field for cracked structures and hence to predict the stress intensity factors along the crack front. The analysis method implemented is based on the theoretical foundations developed for two-dimensional analysis by Portela, Aliabadi and Rooke[2] and for three-dimensional analysis by Mi and Aliabadi[3]. This method has been further developed to include the effect of thermal stresses by Prasad, Aliabadi and Rooke[4] and dell’Erba, Aliabadi and Rooke[5]. In the Dual Boundary Element method, the crack in a structure is represented by special “Dual” elements that allow the stress and displacement fields to be computed on both crack faces without the need to subdivide the body along the crack boundary.

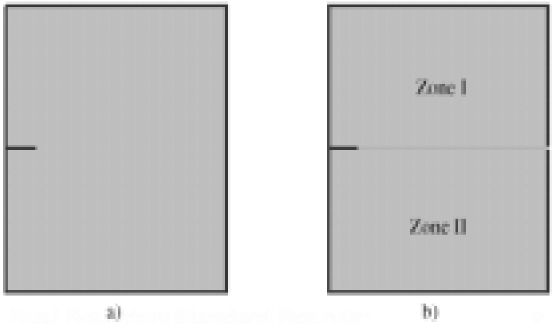


Figure 1 Boundary element models of edge crack. a) Shows the crack modeled using dual boundary elements.

b) Shows the crack modeled using two zones.

The Dual Boundary element method is a powerful solution tool for fracture mechanics, because it is a boundary only representation, high accuracy and the methods ability to represent the high stress fields near the crack front.

Crack Modeling

A crack can be represented in a boundary element model using two main approaches. The traditional approach requires the user to define a zone boundary along the crack surface and continue this through the body of the components being studied. This can be clearly seen in Figure 1b where the problem is split into two zones and the edge crack is extended by a zone interface (shown as a dotted line) across to another external boundary. In the case of an interfacial crack between two materials this modelling strategy is effective and can be used to determine the behaviour of composite laminates for example. This technique while not requiring any special theoretical development places on extra burden on the user for other cases where the interface is an artificial artifact purely required to represent the crack.

Figure 1a shows a model of the edge crack using the Dual Boundary Element Method approach. In this case the modeling is extremely simple and economical. The crack is represented by two elements occupying the same physical location, each element representing a face of the crack.

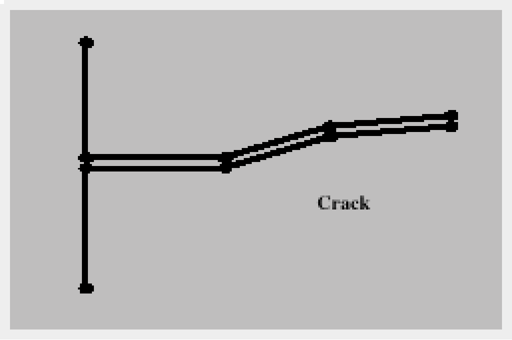


Figure 2 Dual boundary element representation of the crack.

Stress Intensity Factors

Stress intensity factors are computed using both J integral and the crack opening displacement formula. Both Mode I, Mode II and Mode III intensity factors can be computed for edge cracks and embedded cracks. (Note: The approach currently used is based on displacement and stress formulas for computing stress intensity factors for three dimensional problems).

Incremental Crack-Extension Analysis

The incremental crack-extension analysis assumes a piece-wise linear discretisation of the unknown crack path. For each increment of the crack extension, the dual boundary element method is applied to carry out a stress analysis of the cracked structure. The steps of this basic computational cycle, repeatedly executed for any number of crack-extension increments, are summarized as follows:-

- Carry out a stress analysis of the structure;
- Perform stress analysis of model;
- Compute stress intensity factors and crack growth direction;
- Using the crack growth model, the loading spectra and the stress intensity factors, compute the crack growth rate da/dn;
- Step through the loading history computing the new position of the crack front until the crack has changed sufficiently that the stress intensity factors need to be revaluated by the BEM model;
- Modify the model geometry to extend the crack;
- Repeat all the above steps sequentially until a specified number of cycles or crack-extension increments is reached

The length of the crack extension increment may be defined as the result of a compromise between accuracy and computational cost; the smaller the crack increment the more accurate and expensive is the analysis. The results obtained from an incremental crack-extension analysis are a crack path diagram and the distribution of the stress intensity factor variation along the crack path.

Crack-Extension Criterion

Several criteria have been proposed to describe the mixed-mode crack growth. Among them, one of the most commonly used is based on the maximum principal stress at the crack tip, Erdogan & Sih [8]. The maximum principal stress criterion postulates that the growth of the crack will occur in a direction perpendicular to the maximum principal stress. Thus, the local crack-growth direction is determined by the condition that the local shear stress is zero. In practice this requirement often gives a unique direction irrespective of the length of the crack extension increment.

In three dimensional structures the Maximum Principal stress criteria cannot be used so a criteria based on the minimum strain energy density is used along the crack front [9]. This method predicts the crack growth in the direction of region with the minimum strain energy density. The theoretical basis and numerical implementation details are described in [1].

Fatigue Crack Growth Prediction

In practical applications a simple cyclic loading is inadequate to represent the conditions the component or structure will experience during its working life. Therefore the software has been linked to a comprehensive multi-axial loading module which enables real life loading data to be applied to the model. Another important element is the crack growth model that is used to predict the crack growth rate (da/dn). The analysis code allows a range of fatigue growth laws to be represented (for example Paris, NASGRO) and the code is linked the NASGRO database of fatigue crack growth data for fatigue analysis. This also allows the use of retardation models for crack growth[1].

Automatic Remeshing

In the analysis of an embedded crack, at each iteration a series of new elements are added to the crack front. These elements are formed from the positions of the old crack front and the predicted positions of the new crack front.

When looking at an edge crack, the crack can be grown in the same way, with new elements added to model the new portion of the crack surface. However, problems arise at the edge of the crack where the crack intersects the surface of the body. The new predicted positions are based only on the geometry of the crack and the results of the stress analysis itself. The positions are not based on the mesh defined in the model. Therefore the new crack front will not match the mesh defined on the external surface of the body and may not even lie on the surface itself. So, when manually re-meshing the model, the user must initially identify where the crack front intersects the external surface and then the surface itself must be re-meshed.

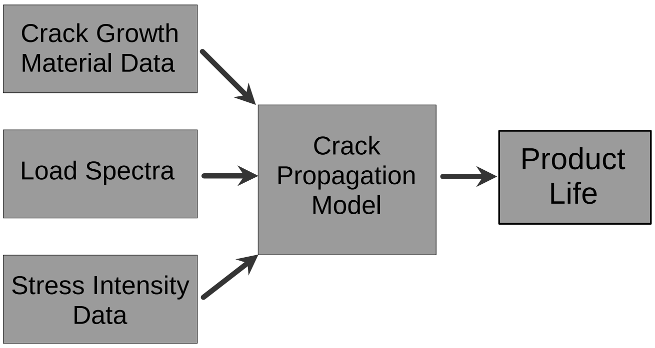


Figure 3 Components necessary for crack growth prediction

In some models the crack re-meshing can be very complex and time constraints and model complexity can prohibit crack growth beyond a few iterations. The aim of the automatic remeshing is to remove this manual work from the user to allow more detailed study of crack growth models to be performed automatically.

In much of the published work on crack growth, the crack growth is performed using embedded or edge crack with simple, regular meshes. However, in many cases the crack breaks out through the geometry surface, and it is not possible to generate regular meshes around the crack breakout region. [3,10]

A new crack growth meshing software has been developed. At each crack growth step, the crack growth model predicts a set of mesh point positions for the new crack front. The meshing program uses these positions to compute a set of new elements on the crack surface. The edges of the new crack front are modified so that they will intersect with the external surface of the body and the external surfaces are also re-meshed in a region close to the crack itself. In addition, as the crack grows then new crack front elements may be split or joined to prevent distorted elements from being produced, thus allowing for a much more accurate computation of the stress intensity factors to be made.

The remeshing of the boundary element model is performed using remesh areas in the model. These remesh areas are collections of elements with similar orientation that can be re-meshed together. These areas are re-meshed using a mapped meshing technique, where the elements are projected onto a flat surface and are then meshed on that surface. Therefore the remesh areas must have similar outward normal directions otherwise the new mesh will be distorted. For example, in a structure such as a cube, with sharp corners, the faces of the box will need to be in different remesh areas.

The algorithm identifies the line where the crack interests with the re-mesh areas and selects elements close to the crack. These elements are then deleted and using the surface generated by the old mesh point, in a flat plane, new elements are created (using an automatic meshing technique) to replace them. Along the crack breakout edge, these new elements are forced to match the edge of the crack itself, in order to optimise the accuracy of the stress solution close to the crack.

Crack Initiation

One important extension of this technique is that it is possible to use the same algorithm to add a crack into a model that has simply been defined to perform a stress analysis. This allows the generation of the boundary element model without having the task of modelling the crack. The crack can then be added anywhere on the model and a new data file automatically generated containing the crack.

This allows greater flexibility in testing different initial positions, sizes and orientations for the crack, without any additional effort in remeshing the boundary element model itself.

Examples 1: Crack Growth in a Circular Bar

In order to explain the methodology for automatic crack growth a simple example is presented which consists of a circular bar under uni-axial loading with a flat central edge crack. The first step in the process is to generate a BEM model of the circular bar sufficient to provide accurate displacements and stresses.

The circular bar has been given a very simple uniaxial loading as shown in Figure 4. The model has been meshed with the same size mesh over the entire model, with no consideration given to where the crack will be initiated.

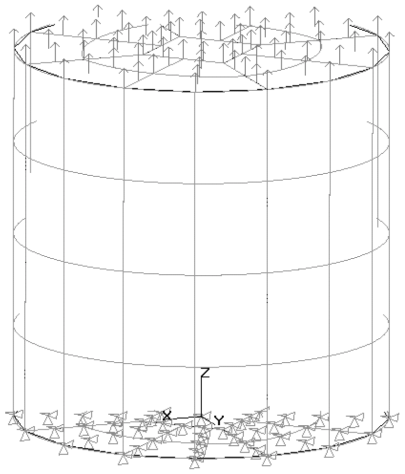


Figure 4 Circular Bar - Base Model

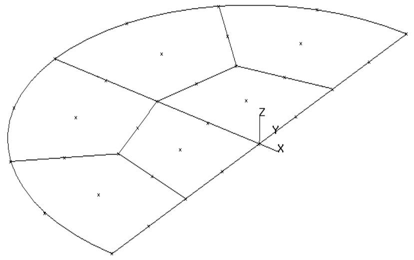


Figure 5 Edge Crack Definition

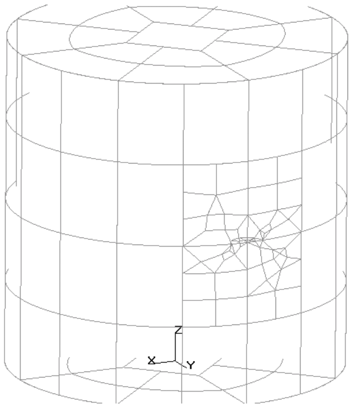


Figure 6 Initial Crack Mesh

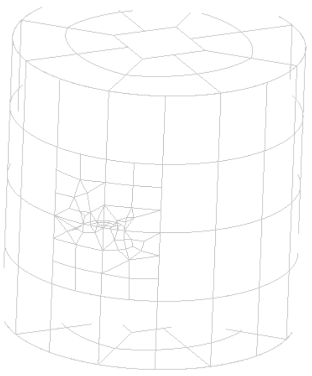


Figure 7 First Crack Growth Step

The model generated using this remeshing program can then be analysed in the usual way. A single analysis is done on this model and a set of new crack front positions is predicted. The crack growth remeshing code can then be re-used in order to generate new elements defining this crack front and to remesh the surfaces around the area of the crack itself. This produces a new geometry for the first crack growth step, as shown in Figure 7**.**

This procedure can be repeated until the crack grows the required distance, number of cycles, stops growing or fails. Here the crack was grown for 5 crack growth steps and the resulting mesh is shown in Figure 8 and the crack surface is shown in Figure 9.

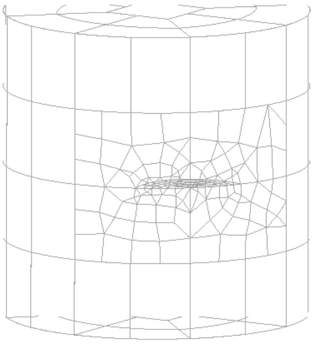


Figure 8 Mesh after 5 Steps

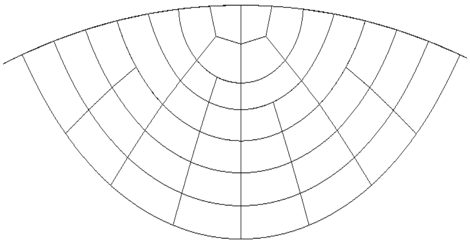


Figure 9 Crack Mesh after 5 Steps

Example 2: Cube under Non-Uniform Loading

In this example a corner crack, show in Figure 10 is initiated into a cube model. Again the cube has been modelled without an initial crack (see Figure 11) and the crack has been added in to give the initial mesh (Figure 12).

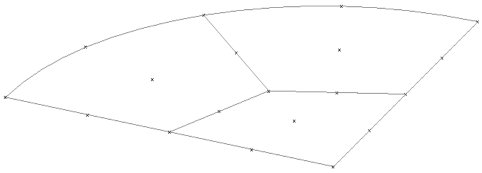


Figure 10 Corner Crack With 3 Elements

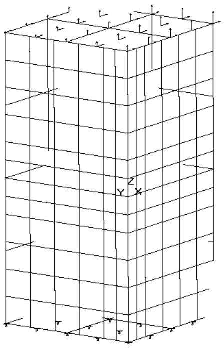


Figure 11 Initial cube Mesh

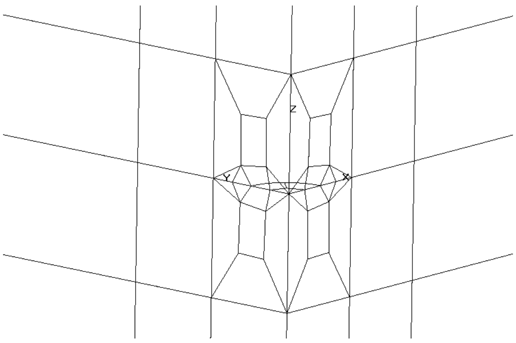


Figure 12 Initial Mesh With Added Crack

In this example, the loading is not uniform, so after a number of crack growth steps (18 steps are shown here), the crack is surface is no longer planar (see Figures 13 and 14).

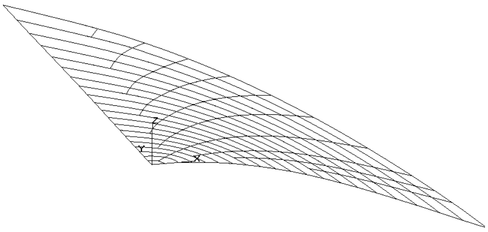


Figure 13 Crack Mesh After 18 Increments

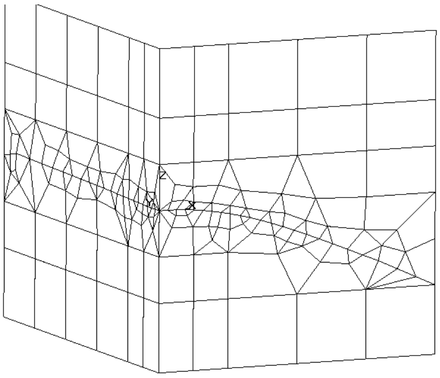


Figure 14 Surface Mesh After 18 Growth Increments

Example 3: Predicting the life of a crank shaft

In this application a crank shaft subject to fatigue loads is analysed to predict how the component will behave once a crack has initiated.

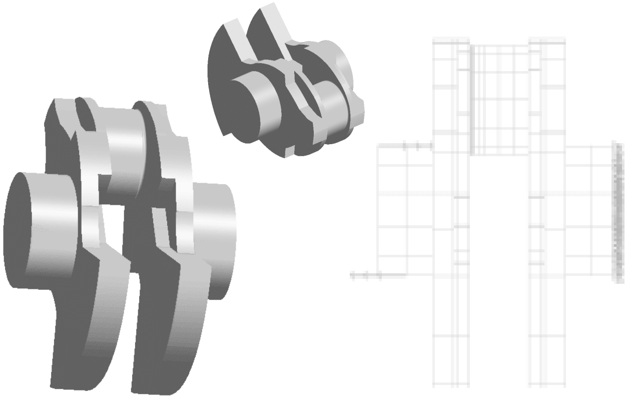


Figure 15 Durability analysis of a crankshaft

The results of the stress analysis suggested that a crack would initiate in the area of a sharp fillet radius. Figure 16 shows how the crack library simplifies the modification of the model to include the crack even in complex geometric regions subject to high stress concentration.

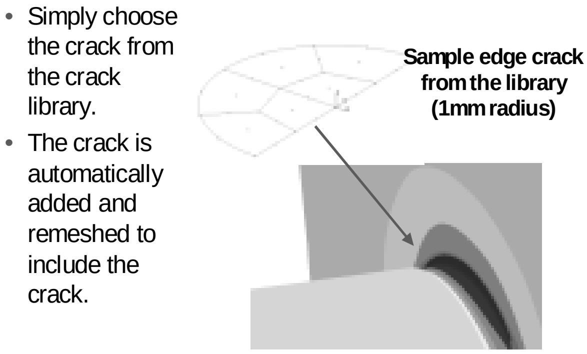


Figure 16 The initial crack is added to the crank shaft

The crank shaft was subjected to a loading spectra which can consist of multi-axial loadings as well as combined mechanical and thermal loads. Figure 17 shows the results of the simulation where the crack has been automatically grown. The direction of growth and shape of the crack can be clearly seen. However, the important design information can be seen in Figure 18 where the crack size variation with the number of loading cycles is shown. Similar information can be displayed for the crank shaft residual strength and Stress Intensity data.

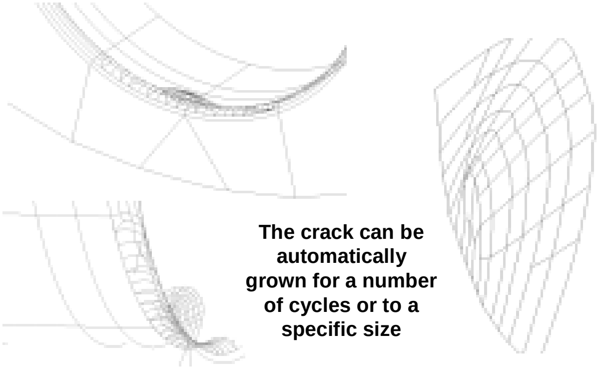
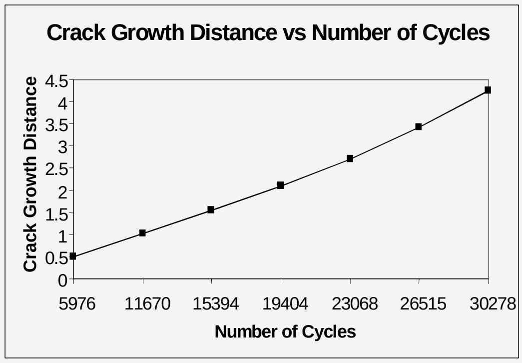


Figure 17 The crank shaft is subjected to the loading history and the behaviour of the crack predicted



At this point the crack has grown 4.5 mm

Figure 18 The key life data can be predicted. In this case the rate of growth of the crack is predicted.

Example 4: Predicting the life of a Gas Turbine Component

In this application a gas turbine component is to be investigated which is subjected to cyclic thermal loads which simulate the operational environment of an aircraft. A crack is predicted to occur in the leading edge of the structure and the design question is will the crack grow and at what rate will the crack grow. Figure 19 shows a solid model of the section to be studied.

Figure 19 Solid view of leading edge of gas turbine section

A small crack is initiated in the leading edge of the component automatically using the crack re-mesher at the specified location. Figure 20. The loading history is applied and the crack grown to obtain the design information on the crack growth rate and the number of cycles required to reach the critical size. This type of geometry presents two difficulties for the re-meshing procedure. The first is that the crack from the crack library requires extensive modification when it is initiated in the model, the second is that the crack approaches an internal surface of the hollow section. See Figure 21. The current algorithm stops at this point and requires the user to make a modification before the automatic process can continue.

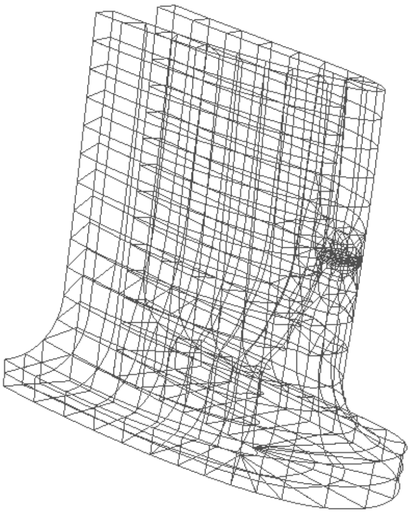


Figure 20 BEM mesh with the crack automatically inserted

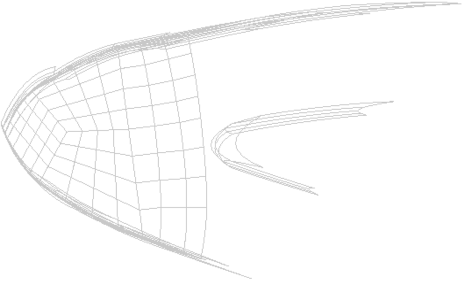


Figure 21 Cross section showing the crack approaching the internal surface

Example 5: Through Crack Modelling

In all the applications shown so far the crack has been only partly through the structure. However cracks which completely penetrate the structure can also be modelled. The following example, demonstrates a three dimensional model of an edge crack in a plate.

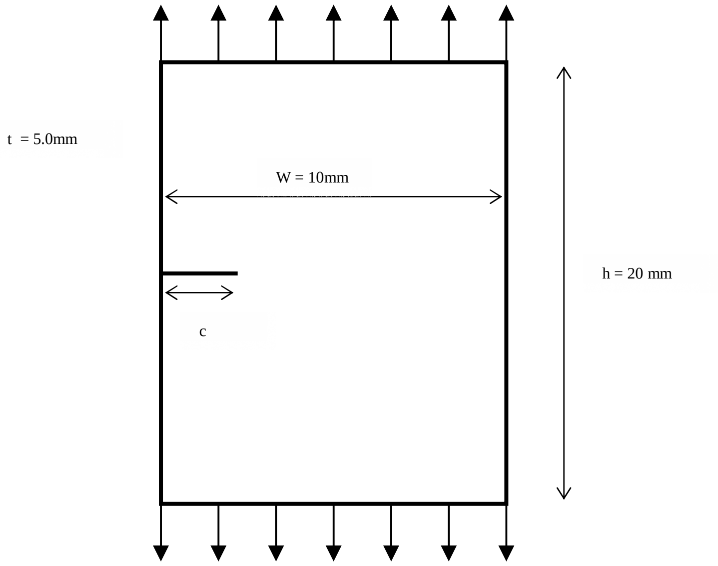


Figure 22 Geometry of Through Crack Model

This model has been generated using a simple block with the dimensions as shown in the Figure 22 above and a through crack is initiated into the model. The crack, as shown below, has a straight crack front and three crack breakout edges that are mapped into the geometry model.

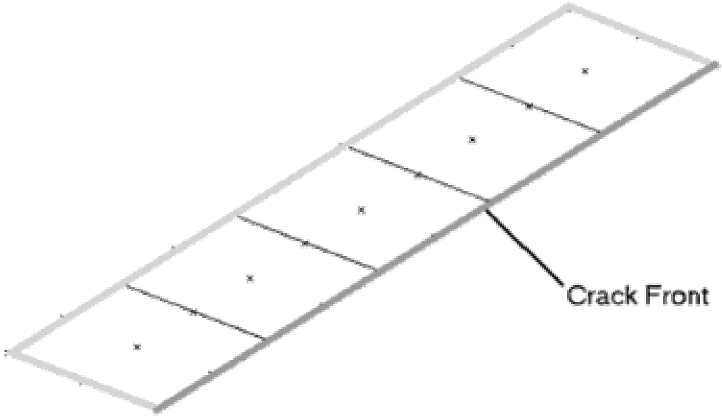


Figure 23 Through Crack - Definition

This model has been compared against results obtained using the NASGRO analysis system[13], using the following crack growth law:

Crack Growth Rate = NASGRO 3 material M2AD11AB1 – NO RETARDATION

This is an aluminium material with crack growth parameter values obtained from the NASGRO3 material database.

A comparison of the SIF values obtained using different lengths of initial crack has been made for both BEASY and NASGRO

Crack Length	BEASY	NASGRO
0.2	0.904	0.9112
0.4	1.252	1.282
0.6	1.577	1.5937
0.8	1.876	1.864
1.0	2.147	2.125

Figure 24 Through Crack SIF Results

This model has been loaded with a cyclic loading with a maximum load value of 200N and a load ratio of value given by R=-0.25. The following graph compares the crack growth rate, as predicted by NASGRO with the crack growth rate computed using BEASY. The NASGRO model predicts 5983 load cycles for the crack to reach a size of 1mm, whereas BEASY predicts 5560 cycles for the crack to reach the same size.

The difference is caused by the slight variation in the stress intensity factors between the NASGRO and BEM model. However as can be seen from these results, the BEM solution provides an accurate result (within 1%), even for the coarse crack definition that has been used in this analysis. The NASGRO solution is based on a solution that does not fully take into account the geometry, boundary conditions and the redistribution of load as the crack grows.

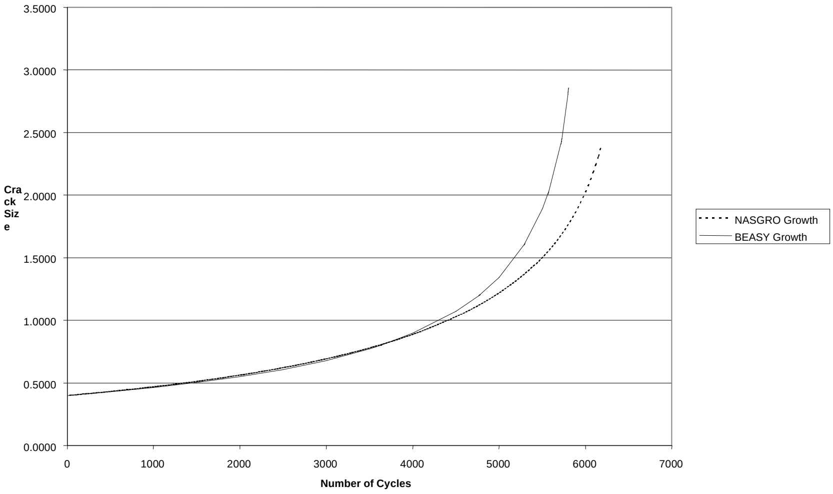


Figure 25 Comparison of Crack Growth Rate

Example 6 Embedded crack growth

In this application the growth of a defect in a railway rail is investigated. In many ways this type of crack growth is simpler than the other examples presented as the crack front is completely embedded. Therefore it is not necessary to remesh the surface as the crack does not penetrate the surface.

. The key elements in the simulation are: -

The pre-stress in the rail;

The crack in the rail and the calculation of the stress intensity factor at the crack front;

The change in the stress intensity factor as the wheel moves along the track.

As a first step a model was generated to simulate the contact between the wheel and the rail. Figure 26 shows the wheel and the rail cross section included in the model. Figure 27 shows the rail model and the mesh on the contact area.

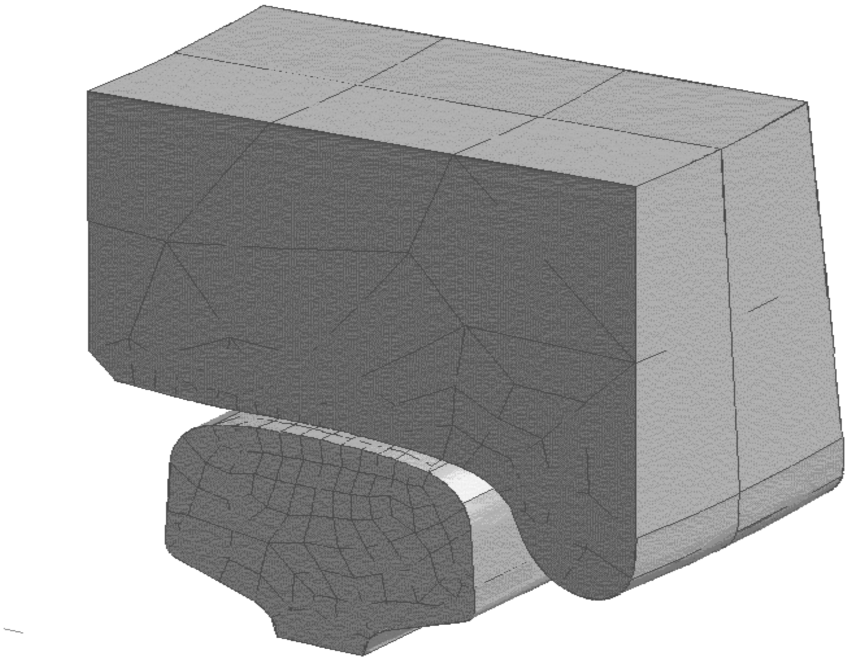


Figure 26 Wheel rail contact model

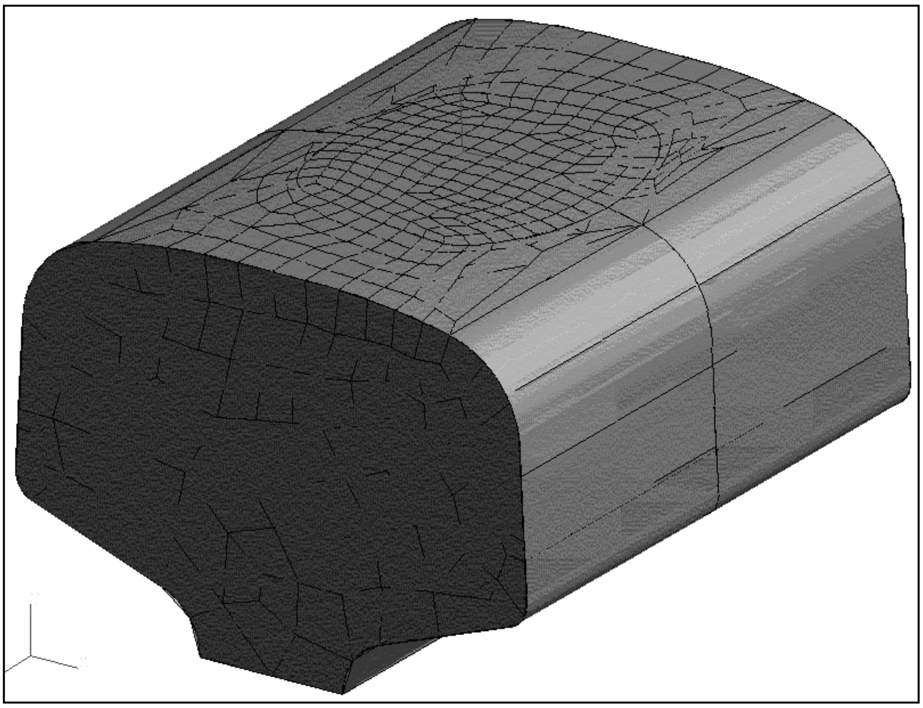


Figure 27 Rail Model showing mesh on contact area

The bottom of the rail was fixed in the vertical and transverse directions. One end of the rail was fixed in the z direction (along the rail). The other end of the rail was loaded with a traction of 200 N/mm**2 to provide a tension in the rail.

An embedded crack was inserted in the rail. Figure 28 shows the location and mesh on the assumed crack. It is clear that positioning of such a crack is very simple and has no effect on the boundary mesh in this case. Thus a study of the effects of different crack size, crack orientation and crack location can easily be performed.

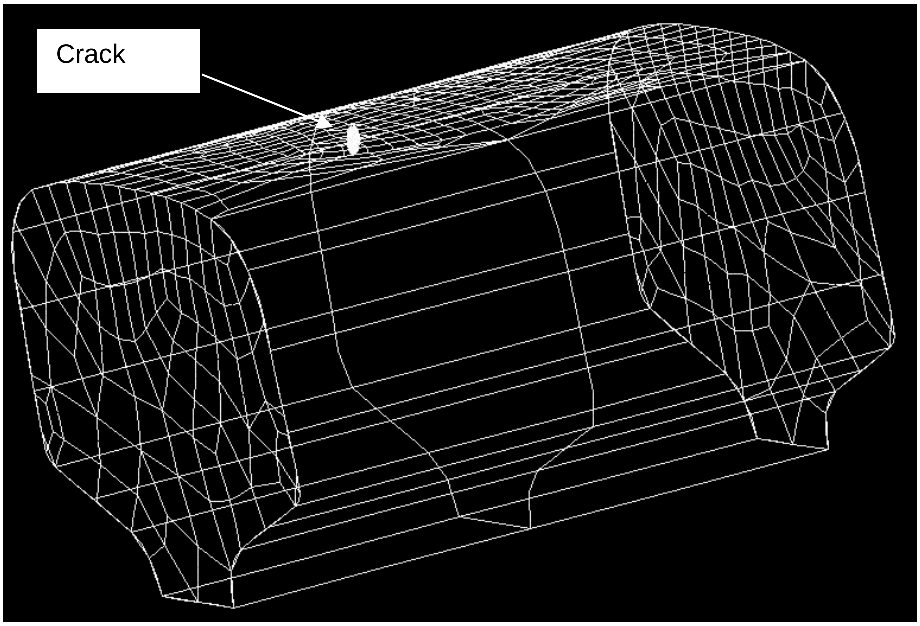
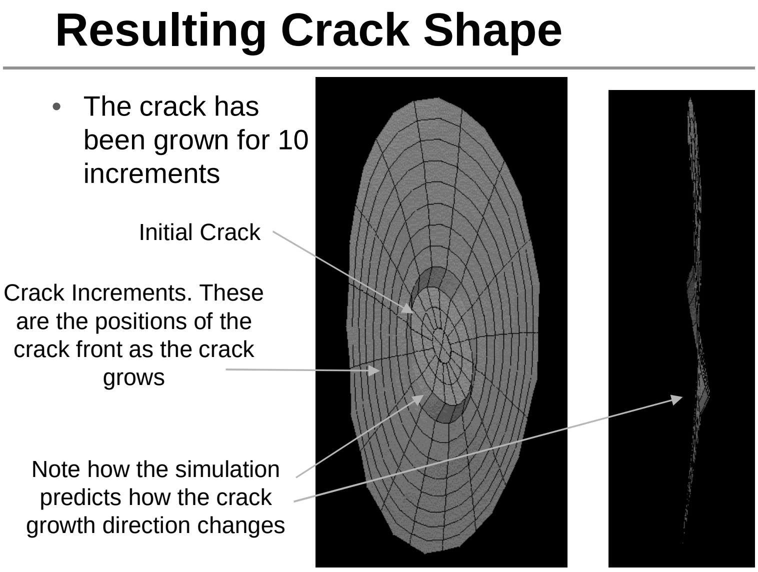


Figure 28 **Rail model showing position of crack**

Under the action of the cyclic load obtained from the contact calculation the growth of the crack was predicted. In this application the orientation of the crack was arbitrarily chosen. Figure 29 shows how the crack grows under the applied loading. In the first step the crack grows in a direction of minimum strain energy, which is indicated in the crack profile changing into a new plane.

In this case the crack was grown a total of ten increments under the cyclic loading. It is also possible to define loading spectra but this was not done in this case.

The results predict how many cycles it requires for the crack to grow to a critical size and the crack growth rate. The mesh is automatically updated at each step to enable the complete process to be performed without interaction from the user.



Resulting Crack Shape

- The crack has been grown for 10 increments

Initial Crack

Crack Increments. These are the positions of the crack front as the crack grows

Note how the simulation predicts how the crack growth direction changes

Figure 29 **Predicted Crack Growth**

Conclusions

A completely automatic approach to predicting crack growth in complex components under thermal and mechanical loads has been presented.

The combination of the boundary element method and automatic surface meshing simplifies the process.

A procedure has been presented to automatically initiate a crack in a model.

The automation of this process fundamentally changes how this type of calculation will be performed in the future. The user effort required to perform a real accurate crack growth calculation is now similar to that required for approximate methods based on standard text book cases.

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