

MSD residual strength assessment for a cracked joint

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Abstract

The present work, realized in the context of BRIT-EURAM (SMAAC) project, summarizes a numerical procedure aimed to evaluation of the residual strength of a cracked lap joint, based on the R-curve analysis and plastic collapse prediction. The model adopted is based on the use of the Dual Boundary Element Method, implemented in the BEASY code, adopting a Linear Elastic Fracture Analysis for SIF's evaluation. Experimental collapse load was available, allowing a comparison with numerical results aimed to validate the described procedure.

1. Introduction

It has been mentioned that stress intensity factors can attain critical values in a way similar to strain energy release rates. The criteria for failure due to unstable crack growth can therefore be written as

$$K_I \geq K_{Ic}, \quad \text{plane strain} \quad (1)$$

or

$$K_I \geq K_c, \quad \text{plane stress} \quad (2)$$

where K_{Ic} and K_c are considered to be constant and are called the *fracture toughness* of the material. It is experimentally found that K_{Ic} is constant for thick sections of a given material. K_c is found to vary with crack length and component geometry and is applicable to thinner sections where stable crack growth can occur.

From equation (1) and (2) the failure criterion can be written as:

$$Y\sigma_c\sqrt{\pi a_c} = K_c \quad (3)$$

where σ_c is the critical stress and a_c is the critical crack length at failure. This equation can be used to assess failure criteria for a component. Hence if a particular crack length is chosen and Y and K_c are both known then

$$\sigma_c = \frac{K_c}{Y\sqrt{\pi a_c}} \quad (4)$$

The critical stress σ_c must not be exceeded by the operating stress if failure of the cracked component is to be avoided. The critical stress will decrease as the crack length becomes longer; this must be considered in the long term assessment of working stresses.

If a stress level σ_c is chosen then the critical crack length is given by

$$a_c = \frac{1}{\pi} \left(\frac{K_c}{Y\sigma_c} \right)^2 \quad (5)$$

The critical crack length must be greater than the minimum detectable crack length a_{\min} so that the component can be inspected for crack growth at regular intervals.

The above criteria do not take into account stable crack growth which can occur in thin sections of some materials. Under these condition the crack will only grow if the load is increasing; if the load is constant the crack will stop. An explanation of stable crack growth was postulated, suggesting that the increase in crack driving force G is initially counterbalanced by the increase in crack growth resistance R under rising load, enabling crack growth to be stable. The instability condition is reached when $G = R$ and $\frac{dG}{da} = \frac{dR}{da}$, i.e. when the curves of G versus crack length and R versus

crack length are tangential to one another. Usually R is expressed in stress intensity factor units, i.e. $K_R = \sqrt{ER}$ and so the instability criterion becomes $K = K_R$, $\frac{dK}{da} = \frac{dK_R}{da}$. R curves have been derived for many materials; more information on R curves and their use can be found in [1-3].

2. Problem and modeling description

The model adopted is based on the usage of Dual Boundary Element Method [4], implemented in the BEASY code [5], adopting a Linear Elastic Fracture Analysis for SIF's evaluation.

The lap joint, proposed for residual strength assessment [6], is represented in Fig.1: it has been modeled by a 2D single plate with a constant traction on one side and constrained in the holes against y-translation in order to model pin actions, whilst no constraints are present in x-direction in order to allow transversal plate shrinkage. With such constraints, longitudinal plate compliance in the overlapping area is neglected whilst it is underestimated in transversal x-direction, introducing an element of approximation; moreover the secondary bending is not considered. The material properties are Young modulus $E=72000 \text{ N/mm}^2$ and Poisson ratio $\nu=0.3$. In the critical cracked area, the pin action modeling has been improved by effectively inserting such pins in the holes and moving the constraints on the pin center. In particular, traction and displacements continuity conditions are imposed on 180 degrees of the pin-hole interface area (the supposed contact area after loading), whilst the remaining part is disconnected by internal spring of negligible stiffness. By means of a convergence study, it has been assessed that it is sufficient to model pins inside cracked holes and those near by (a sufficient number of pins has been modeled in order to get a solution variation of a few percentage units), whilst the remaining holes are just constrained against y-translation, as already mentioned.

Gap elements have also been introduced, to better tackle contact conditions but the solution improvement has been judged quite negligible (less than 2%), except in case of very short cracks emanating from the holes, more sensitive to pin-hole contact conditions. For this reason, and due to the computational effort of a non-linear analysis (causing an enormous increase in run times), they have no longer been used. The J-integral technique is adopted for SIF's evaluation, being more stable than Crack Opening Displacement method, with a variable crack mesh. On the J-integral path, 33 integration points are used (the increment of accuracy with 66 points is completely negligible).

A few minutes of computer run times are needed to run the model on a PC Pentium 200 with 96 Mbyte of RAM and an easy preprocessing phase is allowed by the BEM approach.

The mesh used for the lap-joint is based on about 326 quadratic elements (mesh refinement could be decreased without sensible loss in accuracy): a p-convergence study has been realized showing that cubic elements provide an accuracy improvement of less than 2% and that 2 quadratic elements per 90 degrees are sufficient on the cracked hole, except for very short cracks, where 3 elements are recommendable (possibly with a scaling ratio). It has been proved that, after link up of cracks between holes, there is no longer a load transfer through the pins in the central part of the main crack, with the exception of the extreme ones.

Even in a linear elastic formulation, the SIF's evaluation can be improved by taking into account the elasto-plastic effects, clearly not negligible in a residual strength analysis. This is possible with

the Irwin correction, which suggests to prolong the cracks considered of a virtual quantity $r_y = r_p$, where, for this kind of material, it is possible to assume $r_p = K_{eq}^2 / (S_y^2 * 6.28)$ as a characteristic dimension of the plastic zone at the crack tip [7] (S_y is the yield stress).

Two approaches have been proposed for failure assessment [8]:

- Plastic collapse prediction, based on Von Mises stress exceeding 385 MPa, the average of yield ($S_y=330 \text{ N/mm}^2$) and rupture stress ($S_u=440 \text{ N/mm}^2$), in large zones of ligament;
- R-curve analysis for stable and unstable crack growth assessment.

With reference to the latter, it is well known that the failure criteria for plane strain structure is not valid for the case of thin metal sheet structure, because of extensive slow stable growth (under monotonic loading) prior to instability and catastrophic failure. Here rather than a single material parameter, a material curve (R-curve or K_R -curve), representing an infinity of potential failure points (the crack length at instability is not known a priori), is necessary to make an accurate failure prediction. In this case two criteria must be satisfied to get an unstable crack growth (as already mentioned):

$$K_G \geq K_R \text{ and } dK_G/da \geq dK_R/da. \quad (6)$$

In the R-curve diagram there are two important points:

1. K_0 is the minimum SIF to start the crack propagation;
2. K_c is the critical stress intensity factor (instability point).

K_0 is independent from the specimen thickness and has a constant value for a particular material; on the contrary K_c is strongly influenced from the specimen thickness: thinner specimens give higher K_c values and consequently exhibit slower stable crack growth. A sufficiently thick specimen will result in full plane strain and K_c will then be equal to K_0 .

In order to obtain a crack driving energy (or force) curve an iterative process is needed, which is based on the following steps:

- the load is monotonically increased by little steps and for each of them a linear elastic analysis is performed by a BEM code (BEASY) to get the Stress Intensity Factors (actually when two consecutive configuration have almost equal crack distributions it is possible to avoid the BEASY analysis, imposing a linear variation of SIF's with loads);
- at each step the cracks are prolonged by a length da_i which is provided by the R-curve as a function of the SIF's determined at the previous step; moreover, when the plastic effects become significant, cracks are prolonged by a virtual length r_y (which, in a first approximation, is calculated supposing a linear variation with load of the SIF's worked out at the previous step), in order to provide the Irwin correction for SIF's evaluation;
- for each crack tip, the G curve (crack extension force) is drawn and superimposed to the R-curve in order to find out the instability point, as resulting from the conditions (6);
- during the steady crack propagation some cracks will reach a link-up condition with other cracks or holes, when the plastic zone at the crack tip plus the plastic zone of the approaching crack or hole, is sufficient to cover the remaining ligament.

The residual strength analysis on the lap joint is assessed with reference to the specimen IDMEC-5 [6] modeled in the cracked configuration obtained after 399620 load cycles with the hidden cracks on hole number 1 considered as through cracks (Fig. 2).

The aforementioned procedure gives the following results:

- the first link-up (Fig.4 and Table 1) is obtained with a load of 68 MPa, that is sufficient to create a plastic zone $r_p = K_{eq}^2 / (6.28 * S_y^2)$ covering the ligament between the hole 5 and crack hole 6-left, where K_{eq} is the equivalent Stress Intensity Factor (SIF);
- when a propagating crack reaches a hole we have supposed the nucleation of a small, not detectable crack of length 1 mm on the other side of the hole, as a consequence of local high stress gradients on an aged lap-joint (399620 fatigue load cycles);
- starting from the eighth iteration the Irwin correction has been adopted, because plastic effects become remarkable;
- the second link-up (Fig. 5a and Table 1) is obtained with a load of 143 MPa, between crack hole 10-right (tip 4) and hole 11; there still seems to be some resistant ligament, but just because we have neglected the plastic zone around the hole in diametrical position. As a matter of fact, Von Mises stresses in the ligament are clearly exceeding 385 MPa in a large part of ligament, as it appears in Figg. 5b-c;

- with the same load of 143 MPa in the configuration obtained after the second link-up, from the R-curve analysis, we obtain an unstable crack growth (Fig.6 and Table 2) of tip 3 up to hole 4 and consequent lap joint failure (for plastic collapse). Namely the G-curve becomes tangent to the R-curve reaching higher values and higher gradients, see equation (6), causing an unstable growth of crack 3 (left-hole 5) and consequent failure in the field of fracture mechanics;
- the R-curve (Fig. 7) used is the one available from DASA [6], whose equation is the following (another curve from FAA was available but the point K_0 of initial crack extension has been judged too high):

$$K_R = 18.08 * da^{0.52} - 0.51 * da + 21.49 \quad (7)$$

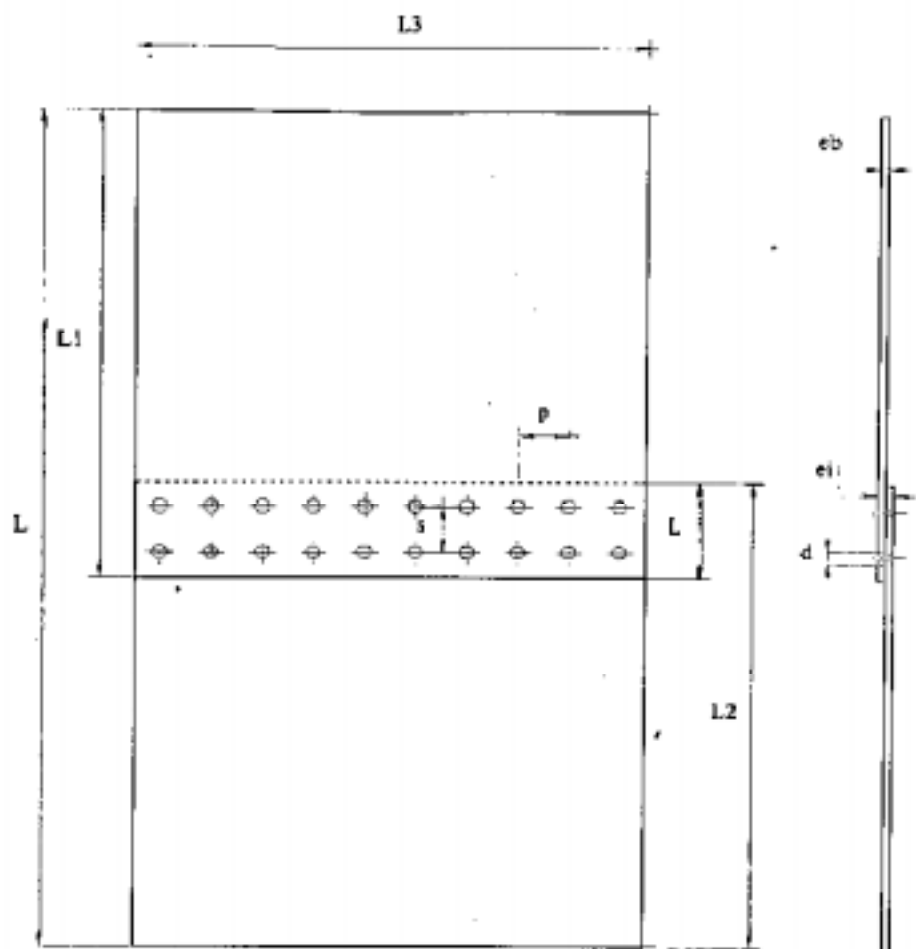
- with K_R in $\text{MPa} \cdot \text{m}^{1/2}$ and da in mm; in this R-curve the Irwin plastic correction is included and, analogously, the SIF's calculated with BEASY are obtained with the Irwin correction;
- the G-curve, superimposed to the R-curve, is obtained in the following way:
 1. crack 3 has been automatically propagated for a certain number of increments in order to get the SIF's for variable crack length;
 2. each SIF is corrected (with the Irwin criteria) by artificially modifying the correspondence between SIF's and related crack increment, in particular by backward shifting the crack length for each step of a quantity r_p .
- from Figg. 8a-b it is evident that with a load of 143 MPa we have not yet reached the plastic collapse (Von Mises stresses are less than 385 MPa in most part of the ligament), in such a way that a failure prediction based on this criteria would turn out to be (slightly) wrong.
- the experimental collapse load is 139 MPa [6], in good agreement with the numerical result.

3. Conclusions

The procedure presented shows a good agreement with experimental results, very attractive run times and an easy pre-processing phase. Some further investigations are necessary to confirm the correctness of the approach adopted, but the results obtained up to now are quite encouraging.

Bibliography

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L	534
L1	300
L2	300
L3	440
Lc	66
ei	3,2
eb	1,6
Number of rows	2 or 3
Number of columns	20
p	22
s	22
d	4,8

Figure 1 - Specimen geometry

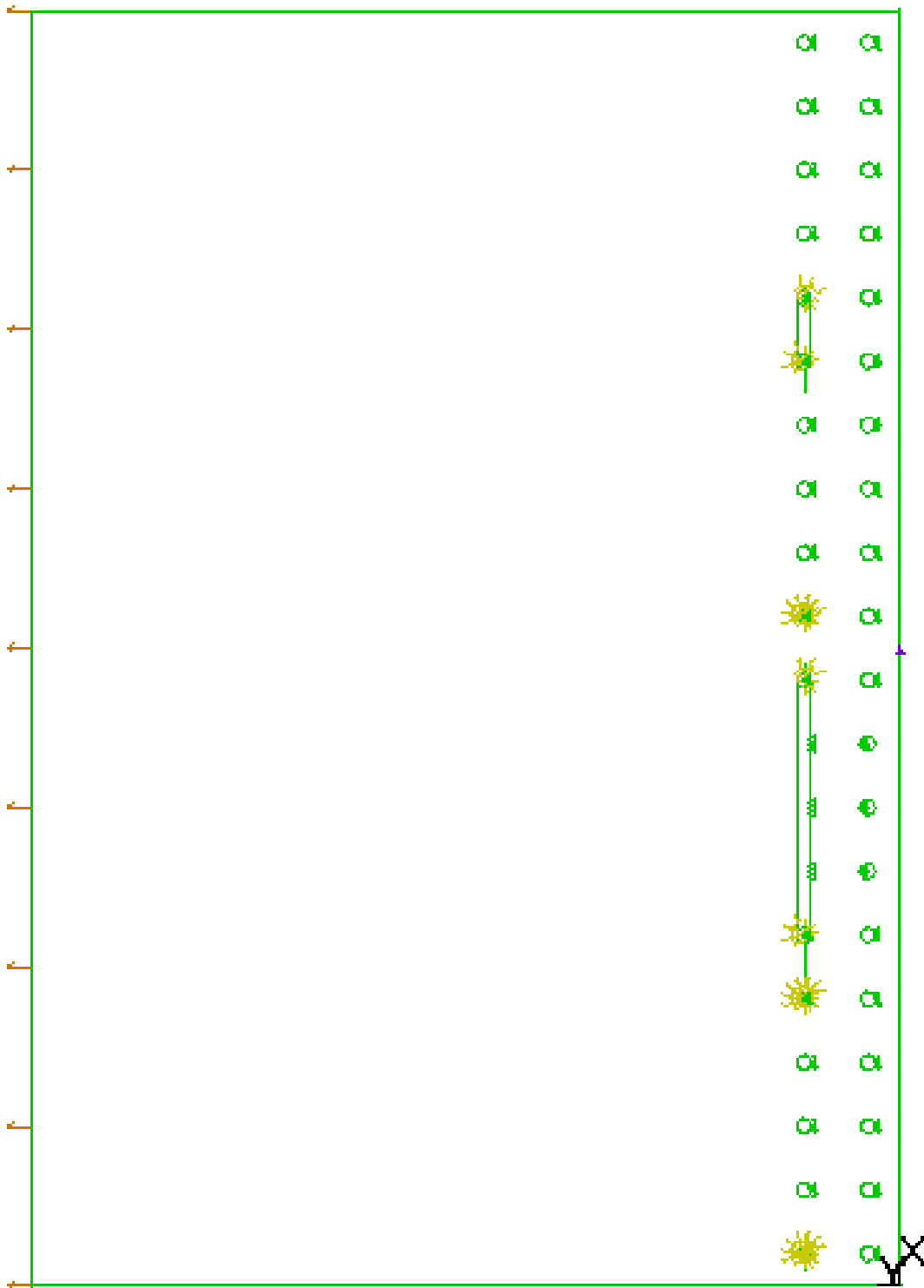


Fig.2: BEASY model of lap-joint in the proposed initial cracked configuration.



Fig 3: Close up of the main crack and domain boundary conditions.

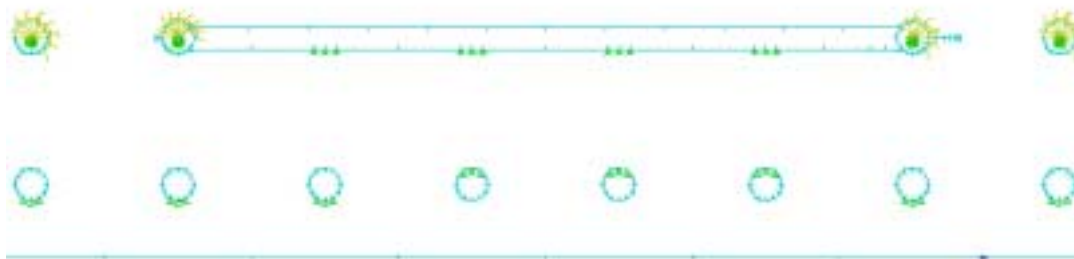


Fig 4: Close up of the main crack after first link-up on the left side.



Fig 5: Close up of the main crack after second link-up on the right side.



Fig. 5b: Internal point position, in the configuration preceding the second link-up.

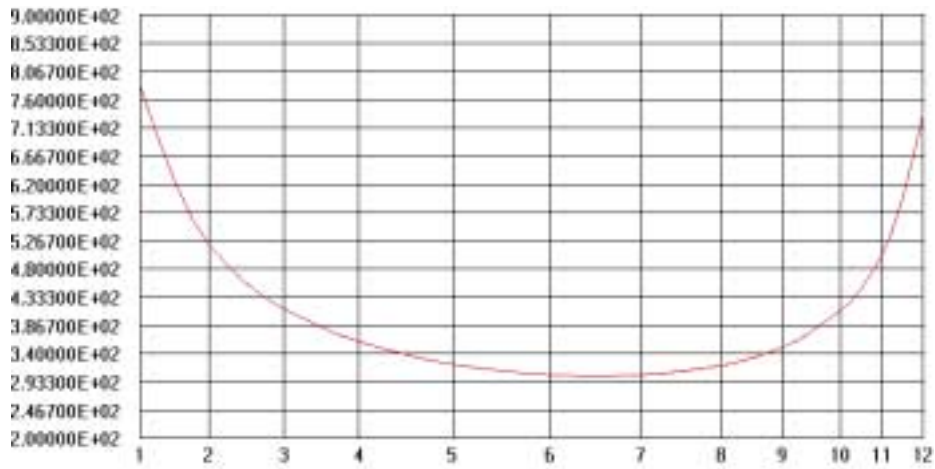


Fig. 5c: Von Mises stresses on the residual ligament of the main crack right side.

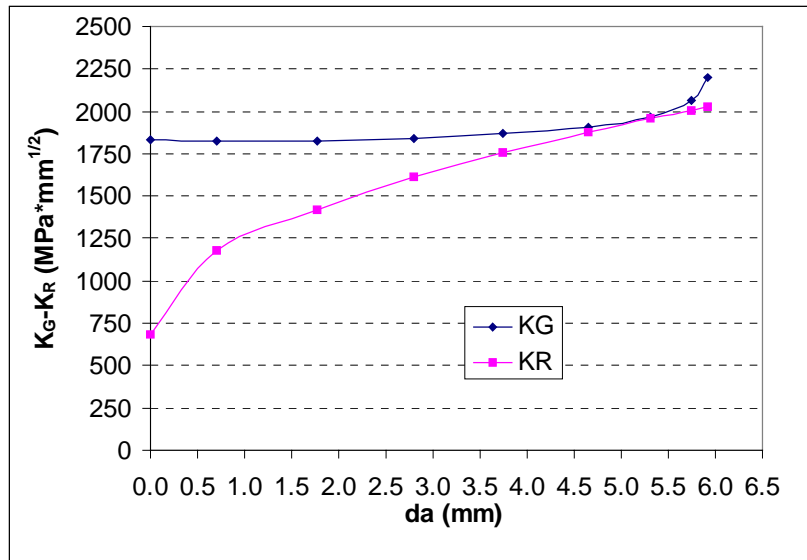


Fig. 6: R-G stability diagram with the G-curve corresponding to a load of 143 MPa.

da _c (mm)	K _G (MPa*mm ^{1/2})	K _R (MPa*mm ^{1/2})
0.0	1829	680
0.7	1823	1176
1.8	1828	1421
2.8	1842	1611
3.7	1870	1755
4.6	1908	1875
5.3	1968	1956
5.7	2063	2006
5.9	2198	2026

Table 2: K_R, K_G data for the crack on left hole 5 (K_G calculated with Irwin correction).

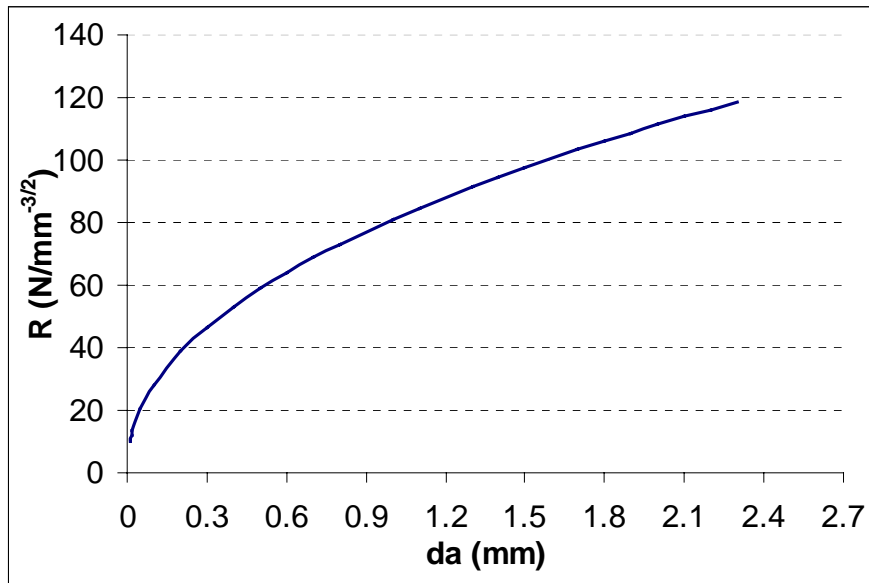


Fig. 7: 2024 T3 R-curve from DASA.

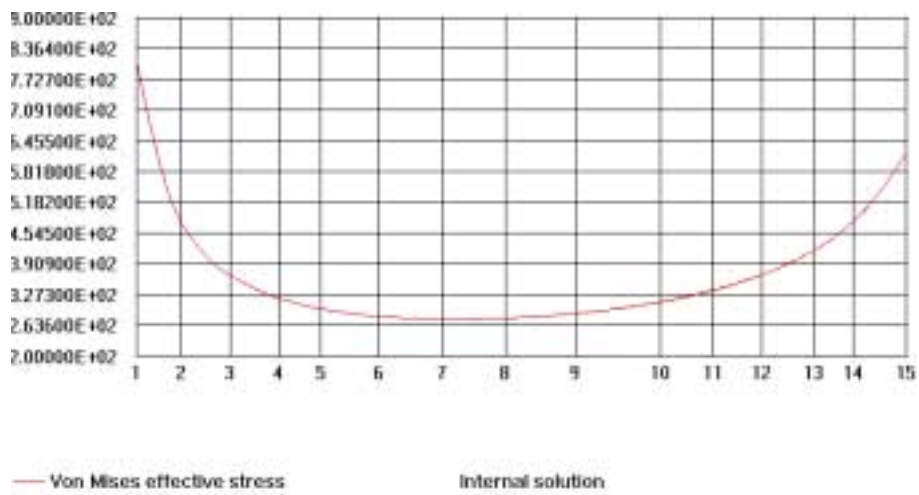


Fig 8a: Stresses in the area between the crack and hole calculated on the internal point depicted.



Fig. 8b: Close-up of the left hole 5 with a load of 143Mpa (note the internal point position).