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Engineering Geology 53 (1999) 25 l-258

ENGINEERING GEOLOGY

Scale dependency in rock strength

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Abstract

It has long been recognised that the size of a rock sample can have a significant impact on the strength recorded of the sample. In this paper the scale dependence caused by the presence of larger weaknesses when the sample volume increases is investigated. An approach based on linear elastic fracture mechanics is proposed to define the behaviour and results are presented. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Rock strength; Scale dependency; Fracture; Modeling

1, Introduction

In any rock sample there are weaknesses present (Fig. 1). These range in size from sub-microscopic and microscopic weaknesses (seventh order discontinuities [1]) of up to millimetres in length to macroscopic weaknesses (sixth order discontinuities) of up to decimetres in length. If the sample size is small it is less likely to include a sixth order discontinuity but the impact of the seventh order discontinuity will be larger due to the reduced ratio of the sample size to the discontinuity size. Consequently if the sample size is larger the impact of the smaller discontinuity will be less (as their size relative to the sample will be smaller) and the larger discontinuity becomes more important as it becomes more probable that they will be present in the sample.

The work will focus on two specific aspects to identify the key parameters which impact the strength of rock samples.

1. How close do discontinuities have to be before significant interaction occurs? That is, at what point does the behaviour of multiple fractures behave differently to that of a single fracture. It is reasonable to assume that the sample size would have no impact if the fractures in the sample behave independently.
2. How large does a discontinuity have to be before the boundary of the sample impacts on its behaviour.

Linear fracture mechanics will be used to develop the theoretical and numerical models presented. The presence of plastic zones near the crack fronts were not considered but they could form the basis of a future study.

2. Linear elastic fracture mechanics

Irwin [2] showed that the stress field in the vicinity of a crack tip was always of the same

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PII: S0013-7952(99)00041-1

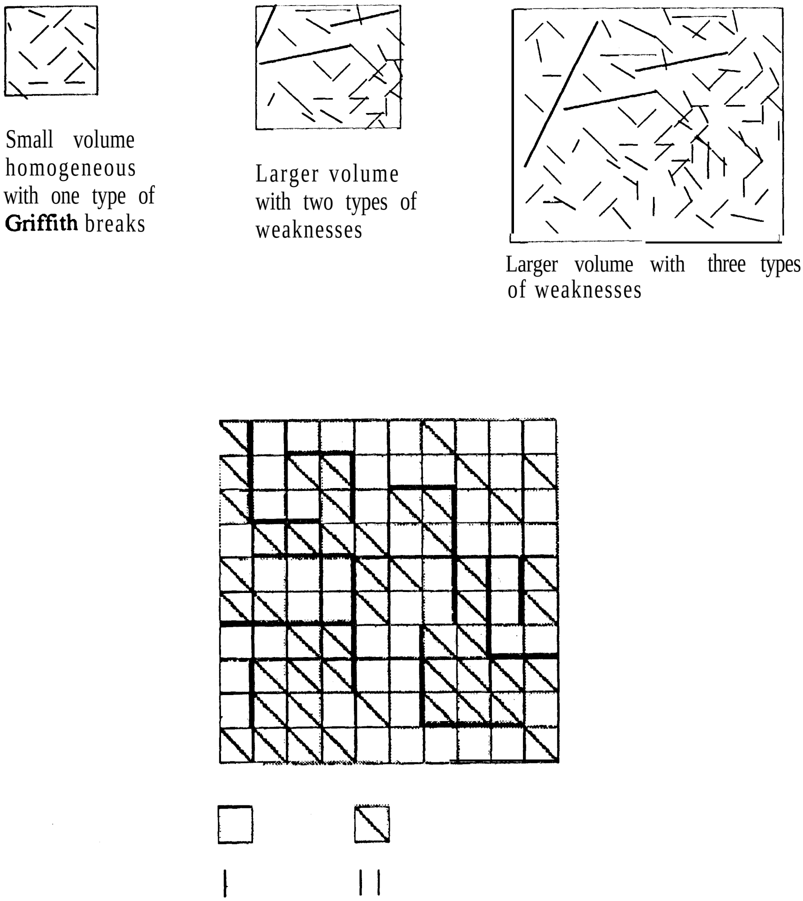


Fig. 1. Presence of weaknesses in rock samples. Upper figures, volume dependence; lower, two-dimensional synthetic crystal matrix made up of two components (I and II) obtained by applying a random selection method. The thick lines represent weak planes formed where sets of at least two components of each type are contacted. For a grain size of 3 mm, the maximum persistence of the weak planes is 12 mm.

form. He showed that the stress field component σ_{ij} at the point (r, θ) near the crack tip is given by:

$$\sigma_{ij}(r, \theta) = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \text{other terms}, \qquad (1)$$

where the origin of the polar coordinates (r, θ) is at the crack tip and f_{ij} contains trigonometric functions. As the coordinate r approaches zero the leading terms of Eq. (1) dominates. The other terms are constant or tend to zero. The constant K in the first term is known as the stress intensity factor.

The stress intensity factor (K) can therefore be used to characterise the stress field near the front of a crack. Two fractures which have identical stress intensities would therefore be expected to behave identically and grow at the same rate.

Stress intensity factors can be obtained from reference databooks for simple geometric cases [3] or using numerical software based on finite element or boundary element techniques. It has been shown that the boundary element method (BEM) is capable of providing more accurate stress intensity factor data and simplifies the process of modelling fractures [4]. Therefore in this study the boundary element method will be used for general geometries and textbook solutions for simple geometries.

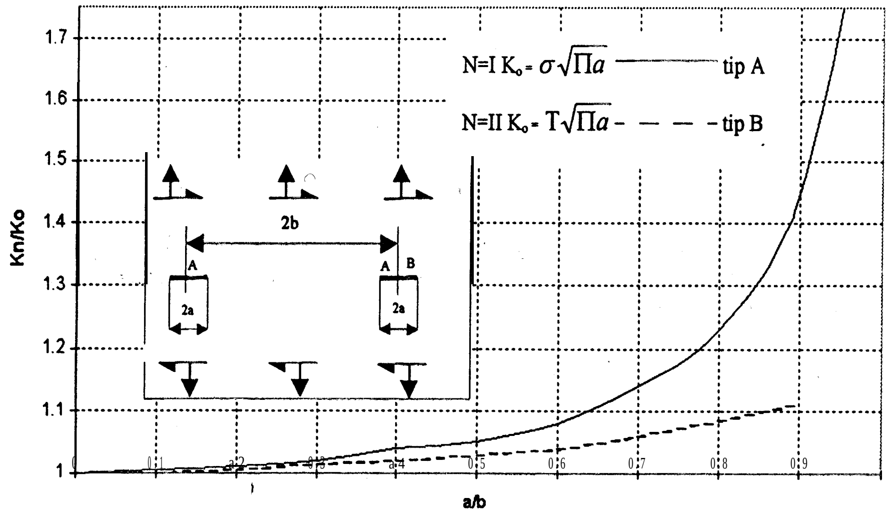


Fig. 2. K_I and K_{II} for two equal length collinear cracks in a sheet subjected to a uniform uniaxial tensile stress or uniform shear stress [3].

3. The boundary element method

The BEM also known as the boundary integral equation (BIE) method is now firmly established in many engineering disciplines as an alterative numerical technique to the finite element method (FEM). The attraction of BEM can be largely attributed to the reduction in the dimensionality of the problem; for two-dimensional problems, only the line-boundary of the domain needs to be discretised into elements and for three-dimensional problems only the surface of the domain needs to be discretised. This means that, compared to a FEM domain type analysis, a boundary analysis results in a substantial reduction in data preparation and a much smaller system of algebraic equations to be solved numerically. Furthermore, this simpler description of the body means that regions of high stress concentration can be modelled more efficiently as the necessary high concentration of grid points is confined to one less dimension.

Another important feature of the boundary element formulation is that it provides a continuous modelling of the interior since no discretisation of the interior is required; this leads to a high resolution of interior stresses and displacements. Other advantages of BEM include the automatic satisfaction of boundary conditions for infinite and semi-infinite domains, thus avoiding the need for numerical discretisation of remote boundaries.

The recent development of the 'dual boundary element method' and automatic crack growth simulation techniques [5] have provided powerful techniques for engineers to predict the behaviour of complex rock structures with multiple fractures. The full theoretical basis for BEM and dual boundary elements is presented in Refs. [4,5].

The software package BEASY is used for the calculations presented in this paper [4].

4. Proximity of discontinuities (fractures)

The impact of the closeness or proximity of fractures to each other is expected to be significant if the fractures are sufficiently close to each other to modify the stress and displacement distribution. To examine any possible orientation of discontinuities would require an infinite number of studies. Therefore the study will focus on a number of regular configurations of fractures. The results for a truly random array of fractures would be expected to fall between these values.

Fig. 2 shows a graph of the stress intensity factor versus the ratio of the crack length to the crack separation distances for collinear separated cracks obtained from Ref. [3]. Data is presented

Table 1
Impact of the proximity of fractures on the K value for collinear cracks

Proximity separation distance/crack length	% Change in K
9	<1
1	5
0.25	23

for two load conditions, tension and shear. The graphs show similar behaviour for both tension and shear loads with a 5% change in K when the separation distance is equal to the crack size. Table 1 summarises the general behaviour.

Therefore significant interaction does not occur until the separation distance and the crack size are approximately equal.

Fig. 3(a and b) shows a series of graphs of the stress intensity factors for offset parallel cracks. It is not possible to present the data in the same way as both the horizontal and vertical separations are important.

Significant interaction does not occur until the a/b ratio exceeds 0.4 (e.g. when distance separating the cracks is approaching the crack length). The maximum interaction occurs when the two cracks have the same x coordinate (i.e. tip A and tip A) but are separated vertically [see Fig. 3(a)]. However, the impact decreases to $<20\%$ when the crack separation distance vertically is equal to the crack length.

In summary no significant crack interaction occurs unless the distance separating the discontinuity is less than the length of the discontinuity.

Therefore from a statistical point of view if the density of the fractures is such that the average separation distance between the fractures is greater than the average fracture size the behaviour of the sample is not expected to be modified by the interaction of the fractures.

5. Interaction of discontinuity and sample boundaries

The rock is assumed to be made from a continuum of material with a series of fractures present. The properties of the contiuum do not change with sample size (within the parameters of this study). Therefore any change in the behaviour of the sample must be due to the presence of the fractures.

If the conditions under which the change in behaviour of the fractures (due to change in the sample size) is identified, then this provides the information necessary to predict the impact of sample size on the strength of the sample.

Again it is not feasible to consider a completely random array of fractures, although it is possible to model any distribution (Fig. 4). Therefore a two-dimensional model with a single crack is studied (Fig. 5).

If the sample size is large (compared with the crack) the crack will behave as if it is in an infinite sample. If the sample size is reduced there will become a point when the behaviour of the fracture will change due to the proximity of the boundary of the sample.

Series of numerical studies were performed using the BEASY software [6] on cracks of different size in a sample plate. Figs. 5 and 6 shows the BEASY model used.

The results show that if the fracture size is $<20\%$ of the size of the sample, the impact on the K value is $<10\%$. At 10% the impact is $<2\%$. Therefore if the average size of the fracture is less than one tenth of the size of the sample, the scale dependency of the sample is negligible (note -- the shape of the sample can also be significant but this has not been considered here).

6. Proximity of cracks to boundary of the sample

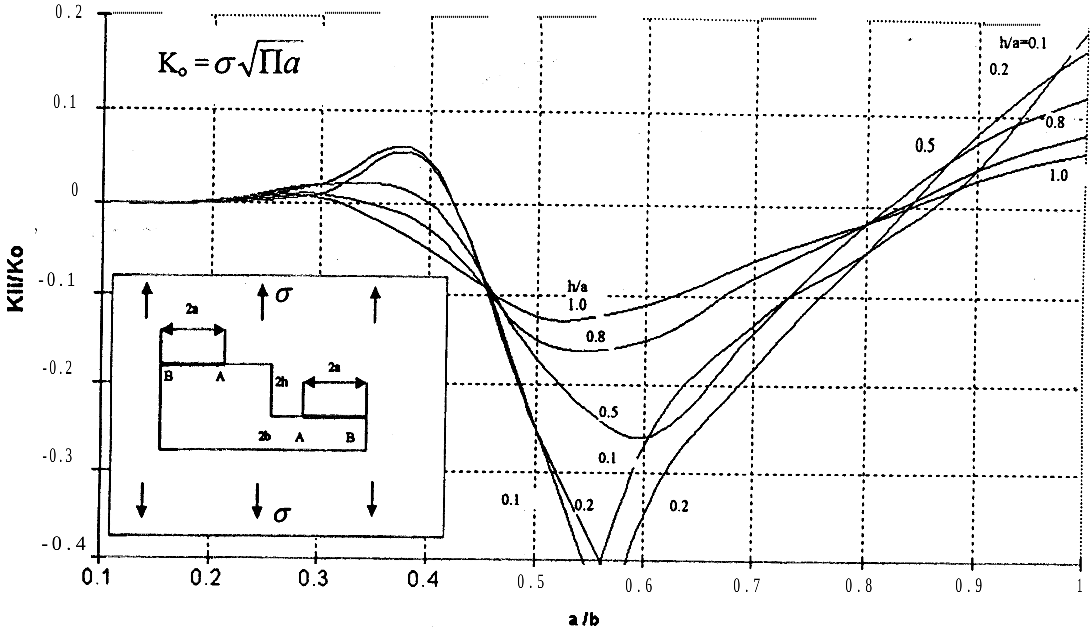
The proximity of the cracks to the boundary as shown above affect the behaviour of the specimen. However the number of cracks near the boundary compared with the total number of cracks in the specimen will also be important.

Consider a square two-dimensional specimen, size $b \times b$ and a crack size of a.

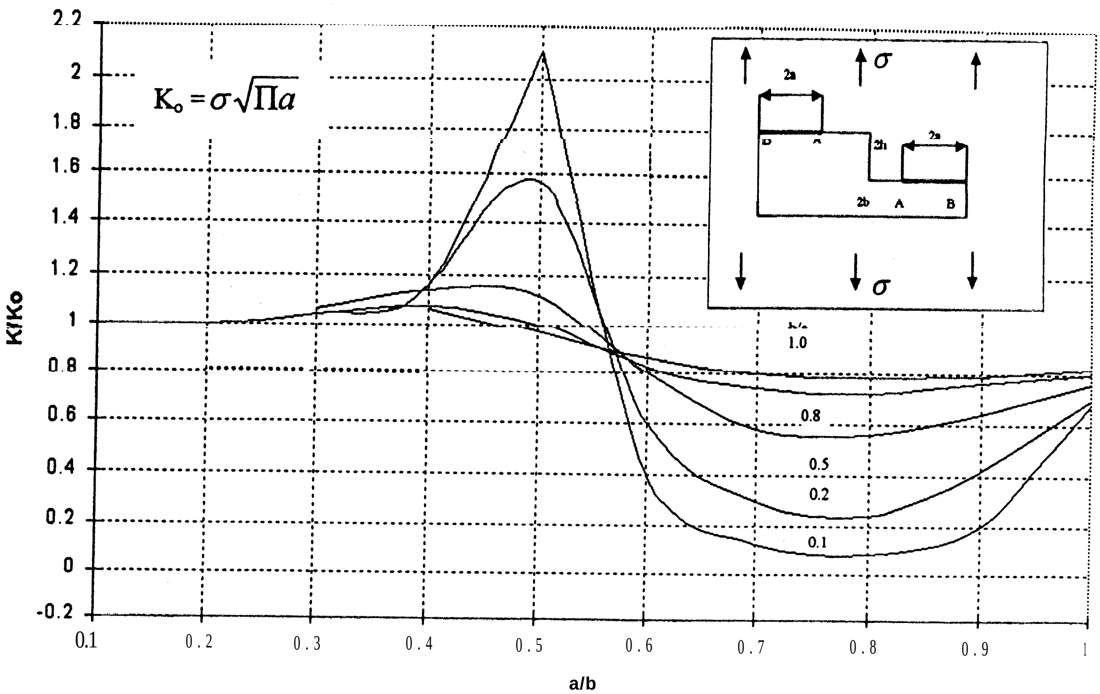
Total area of the specimen $A = b \times b$.

Assume the crack is not affected by the boundary if the distance to the boundary is $>5a$.

Therefore the total area of specimen less than



0 a



(b)

Fig. 3. (a) K_{II} for tip A of two equal length offset parallel cracks in a sheet subjected to a uniform uniaxial tensile stress K_I for tip A of two equal length offset parallel cracks in a sheet subjected to a uniform uniaxial tensile stress.

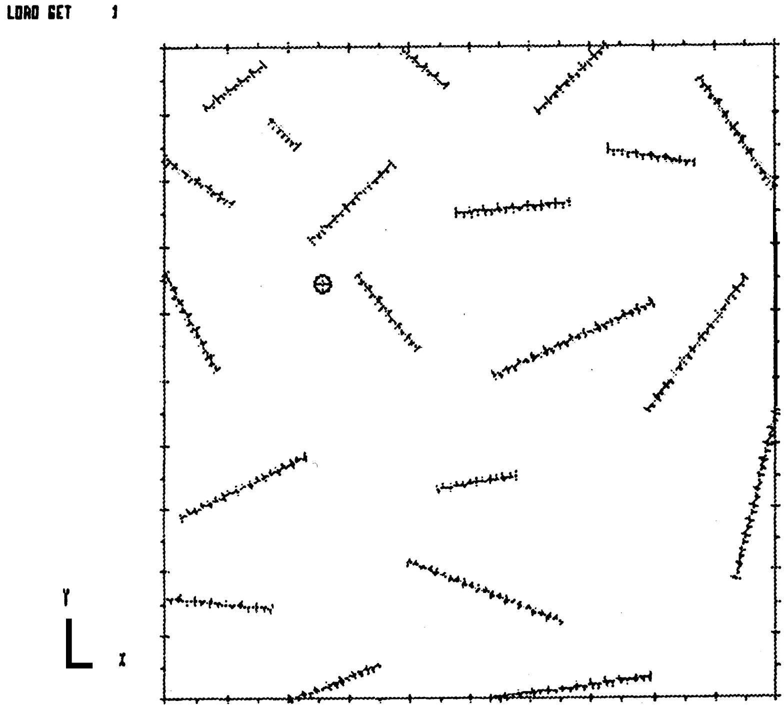


Fig. 4. Random array of fractures modelled using BEASY.

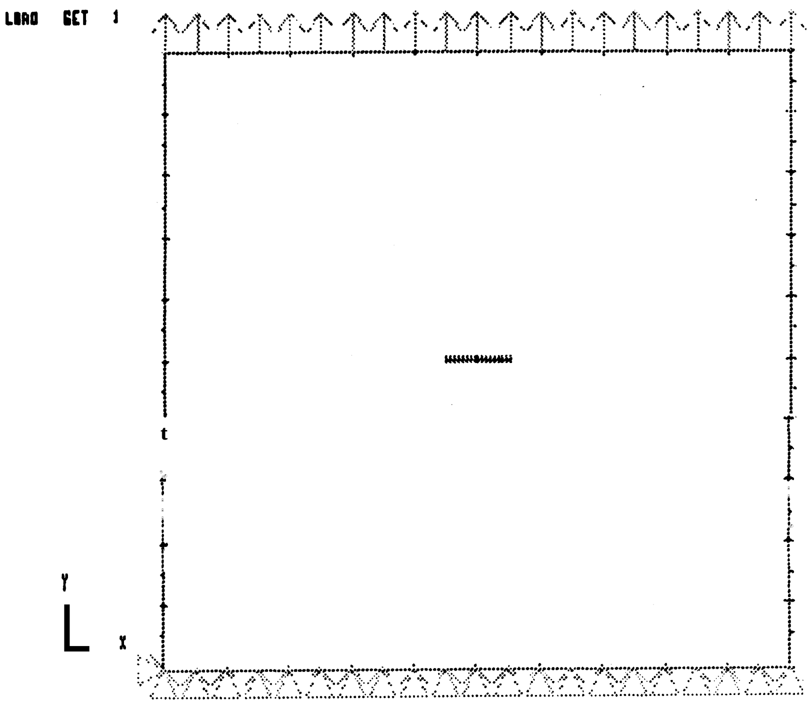


Fig. 5. Sample with single fracture modelled using BEASY.

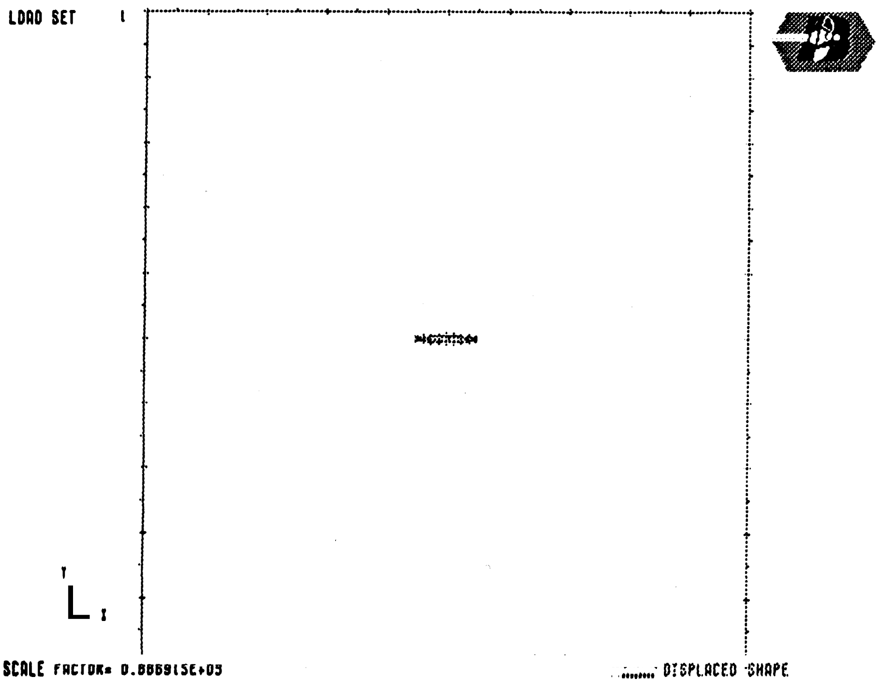


Fig. 6. BEASY model of central crack in a plate showing deformed shape.

the distance $5a$ from the boundary is:

$$4(b-5a)5a. \tag{2}$$

Expressing the ratio of fractures affected by the proximity to the boundary as a percentage of the overall sample size.

$$\text{Ratio} = \left[\frac{4(b-5a)}{b^2}\, 5a\right]100 \tag{3}$$

$$\text{Ratio} = \left[20\,\frac{a}{b} - 100\left(\frac{a}{b}\right)^2\right]100. \tag{4}$$

Applying Eq. (4) reveals results shown in Table 2.

Combining the above data with the stress intensity data should provide a methodology for quantifying the scale dependency of sample size.

Table 2
Percentage of fractures affected by boundary

Fracture size a	Sample size b	Percentage of fractures affected by boundary (%)
1	10	100
1	15	89
1	20	75
1	100	20

7. Conclusion

A basis of methodology for predicting the impact of sample size on the strength of rock has been established.

Data has been developed quantifying the impact of the proximity of the sample boundary to fractures in the sample and the conditions necessary for inter fracture interaction. The techniques based on linear fracture mechanics should be extendable to any geometry of fracture or sample.

For common cases (e.g. the rock structure shown in the lower section of Fig. 1) the effect of the proximity of the weaknesses gives a change in K of > 20% from the case with sparse discontinuities. Further, the percentage of fractures affected by the boundary is 100% in this case as can be concluded by using the results from Tables 1 and 2.

The results presented are for two dimensions but can easily be extended to three dimensions.

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