BEM for Modelling Cathodic Protection Systems in Multi-Layer Electrolytes

Industrial Applications in Well Casing Structures

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Outline

- Introduction to CP systems & corrosion control
- Modelling of CP systems with BEM
- Multi-layer BEM for stratified electrolytes
- Case Study (Well casings)
- Analysis of results
- Conclusions
Examples of CP Systems for Corrosion Control

- Offshore Jacket structures
- Reinforced Concrete structures
- Tank bases
- Defence: Corrosion & Signatures
- Retrofitting
- Wells casings
- Galvanic Corrosion
- Ships, vessels, FPSO structures
- Coatings
- Aircrafts

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**Current Density, Longitudinal Component**

<table>
<thead>
<tr>
<th>Current Density (Initial)</th>
<th>Current Density (Optimised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>5.00E-02</td>
<td>5.00E-02</td>
</tr>
<tr>
<td>1.00E-01</td>
<td>1.00E-01</td>
</tr>
<tr>
<td>1.50E-01</td>
<td>1.50E-01</td>
</tr>
<tr>
<td>2.00E-01</td>
<td>2.00E-01</td>
</tr>
<tr>
<td>-1.00E-01</td>
<td>-1.00E-01</td>
</tr>
<tr>
<td>-1.50E-01</td>
<td>-1.50E-01</td>
</tr>
<tr>
<td>-2.00E-01</td>
<td>-2.00E-01</td>
</tr>
<tr>
<td>-2.50E-01</td>
<td>-2.50E-01</td>
</tr>
</tbody>
</table>

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**Defence: Corrosion & Signatures**

- Ships, vessels, FPSO structures

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**Ships, Vessels, FPSO Structures**

- **Galvanic Corrosion**
- **Coatings**
- **Well casings**
- **Defence: Corrosion & Signatures**

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**Conductivity [S/m]**

- 0.02
- 0.10
- 0.50
- 0.67
- 1.00
- 0.001
- 0.50

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**Well casings**

- **Defence: Corrosion & Signatures**
- **Ships, vessels, FPSO structures**
- **Aircrafts**
Examples of CP Systems for Corrosion Control

- Offshore Jacket structures
- Reinforced Concrete structures
- Tank bases
- Defence: Corrosion & Signatures
- Retrofiting
- Pipelines
- Coatings
- Well casings
- Galvanic Corrosion
- Ships, vessels, FPSO structures
- Aircrafts
CP SYSTEMS FOR WELL CASINGS

Reference electrode

Coating (BF, ρ)

Coating defects

Polarisation (boundary condition)

Electrolyte

Current flow

Power supply

Ground-bed anode (Polarising BC)

Anodic Polarisation (boundary condition)
INTRODUCTION

- Computational modelling of CP systems
  - Predict lifetime of the CP system
  - Optimise the corrosion control technique
  - Provide improved interpretation of data from field surveys
  - Facilitate the planning of cost-efficient field surveys
  - Helps users extract more information from field data
  - Correlates and verifies the consistency of observed measurements
  - Detect potential anomalies in the performance of the CP system (i.e. stray currents)
Governing equations

**Mass flux**

\[ J_k = -D_k \nabla c_k - \frac{z_k F}{R_0 T} c_k \nabla \varphi \]

**Mass conservation**

\[ \frac{\partial c_k}{\partial t} + \nabla \cdot J_k = \rho_k \]

**Electrolyte Current**

\[ i = F \sum_k z_k J_k \]

**Electro-neutrality**

\[ \nabla \cdot (\sum_k z_k J_k) = 0 \]
**Governing equations**

- **Bulk of the electrolyte:**
  \[ \nabla \cdot \left[ -k(x) \nabla \varphi(x) \right] = p(x) \n\]

- The polarisation in the electrodes is described by non-linear relationships between current density and potential difference given by the Butler-Volmer equation, or experimentally determined.

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**UNCLAD AL2024 - POLARISATION CURVES**

- **Potential [V]**
- **Current density [A/m²]**

Potential at zero current

Pitting potential

AL2024-5010
AL2024-9700
AL2024-49300
CFRP
BEM Conceptual model

Cathode $\Gamma_c$
Non-linear boundary condition prescribed by:
$V_e - V_m = f_c(j)$ and finite electric resistance in the metal

Electrolyte ($\Omega_e$)
- Multi-layered
- Semi-infinite or bounded

Insulating boundary

Anode array
Non-linear boundary condition prescribed by:
$V_e - V_m = f_A(j)$

NonLinear BEM

Linear FD
Kirchhoff circuit's equations
Discrete and distributed resistors

Return path circuit
Modelling with BEM

Conservation of charge in the electrolyte + Ohms law leads to:

\[
\nabla \cdot \left[ -k \nabla V_e(x) \right] = 0
\]

The Boundary Integral formulation of the above equation leads to:

\[
c V_e(x) + \int_{\Gamma} \frac{\partial G(x,y)}{\partial n} V_e(y) d\Gamma(y) - \int_{\Gamma} G(x,y) \frac{\partial V_e(y)}{\partial n} d\Gamma(y) = 0
\]
Green’s Function for Multi-layer media

- **STEPS**
  - Solve the Laplace equation for a single layer in the spectral domain
    \[ G(\gamma, z, m, n) = \frac{A_{mn}e^{-\kappa} + B_{mn}e^{\kappa}}{2\sigma_m \gamma} \]
  - Impose boundary conditions of continuity of flux and potential across the different layers in the spectral domain
  - The solution will be a linear combination of the form:
    \[ \sum_{k=1}^{4} \frac{C_{kmm}(\gamma)}{2\sigma_m \gamma} e^{\pm(D_{mk} \gamma z)} \]
  - Transform the solution obtained in the spectral domain to the space-domain
Multi-layer Kernel

In the end the approach for obtaining resembles a Method of weighted Images:

\[ G(x_i, x_j, m, n) = \frac{1}{4\pi\sigma_m} \sum_{k=1}^{N\exp} \alpha_{ijml} \frac{\alpha_{ijml}}{|x_i - x_j + g_{ij}|} \]
MULTI-LAYER ZONE CALCULATION

BEM Kernel: \[ G(x_i, x_j, m, n) = \frac{1}{4\pi\sigma_m} \sum_{k=1}^{N} \exp \left( \frac{\alpha_{ijml}}{\|x_i - x_j + g_{ij}\|} \right) \]

Goals:
1- Eliminate DOF at the interfaces between regions of dissimilar properties
2- Solve extremely thin electrolytes without compromising size elements
3- Minimise CAD modelling work
4- Improve accuracy for DOF near interfaces

Layered non-homogeneous soil
CP and Pipeline Network

Typical pipeline network configuration
In particular, the condition \( h << L \) can be efficiently handled by the software.
Side effects

- The number of degrees of freedom becomes independent of the number of layers, thus speeding up the solving time.
- The calculation time of the influence matrices increases with the number of exponential terms used to represent the multi-layer Green’s function.
- The multi-layer Green’s function contains the Green’s function of the homogeneous Laplace equation \((1/r)\) in the particular case of one layer extending to infinity in all directions.
From Multi-Region to Multi-Layer Modelling: Application to Modelling of CP in Well Casings
Case Study

- The anode bed, represented by a 30cm diameter by 8 m long cylinder, is located 75 m apart from the well in x direction.

- The top of the anode is 30m deep and the total current injected into the CP system is 10 Amps.

- Conductivity values:
  - $\sigma_1 = 0.1$
  - $\sigma_2 = 0.5$
  - $\sigma_3 = 1.0$
  - $\sigma_4 = 0.5$
  - $\sigma_5 = 0.67$
  - $\sigma_6 = 0.001$
  - $\sigma_7 = 0.02$
Case Study

The goal is to ensure that:

- the normal component of the current density on the steel is always positive (i.e. flowing from soil to the structure)
- the polarisation potential on the steel is more negative than a certain value (-800mV relative to Ag:AgCl) and no more negative than -1300 mV
- the power delivered to the system is minimal
- anomalies occurring below ground level can be correlated with potentials measured at ground level
Case Study

- The conceptual model consists of a well casing and one anodic ground-bed in stratified soil consisting of 7 layers.
- The well casing is 1750m deep and consists of four sections of different diameter.

<table>
<thead>
<tr>
<th>Pipe section</th>
<th>Soil layer</th>
<th>Span Zmin</th>
<th>Span Zmax</th>
<th>External diameter</th>
<th>Wall thickness</th>
<th>Resistance per unit length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>300</td>
<td>0.35</td>
<td>0.012</td>
<td>1.36E-05</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>300</td>
<td>500</td>
<td>0.25</td>
<td>0.012</td>
<td>1.94E-05</td>
</tr>
<tr>
<td>3</td>
<td>2,3,4</td>
<td>500</td>
<td>800</td>
<td>0.175</td>
<td>0.012</td>
<td>2.8E-05</td>
</tr>
<tr>
<td>4</td>
<td>4,5,6,7</td>
<td>1200</td>
<td>1750</td>
<td>0.15</td>
<td>0.012</td>
<td>3.34E-05</td>
</tr>
</tbody>
</table>
Design Case Scenarios

- **OBJECTIVE:** To characterise the influence of the environmental conditions (soil properties) on the CP system

  - **“BARE”**
    - The casing is assumed to be in direct contact with the electrolyte
  
  - **“WET”**
    - The metallic case is inside a cement cylinder which is considered highly porous and water has infiltrated.

  - **“DRY”**
    - Same as wet, but with lower conductivity.

  - **“MIX1”**
    - This scenario assumes that all the casing sections are inside a cylindrical column of dry cement with the exception that in the area of layer 3, the column is considered to be highly porous and therefore the conditions of the wet case apply.
Polarisation curves considering steel in concrete with varying water saturation

- The concrete introduces an additional ohmic resistance to the polarisation curve characterised by:

\[
j(r_2) = \frac{k_c}{r_2} \ln\left(\frac{r_1}{r_2}\right) \Delta \phi
\]

\[
\rho_s = \frac{r_2}{k_c} \log\left(\frac{r_1}{r_2}\right)
\]
Design Case Scenarios

WET

DRY

MIXED

Sketches are not to scale
It can be seen that there is an approximate 300mV shift between the “Dry” and the “Wet” cement condition over the majority of the casing.

However in the deep layer where the conductivity is very low inadequate protection is obtained for all the scenarios considered.
Results: Profiles along well casing
The predictions from the model for the current density are similar to that obtained from a CPET Log. Using the model, a number of “virtual CPET logs” can be quickly predicted to test the robustness of the design and to investigate solutions to potential problems.
Design Optimization

- One option could be to investigate if increasing the anode current will provide improved protection.

- For the MIX case, the graphs clearly show the limited impact of increasing the ICCP current on the under protected area of the casing.
Predicted Current Flow To The Casing

- The model allows us to understand where in the structure the extra current provided by the CP system is going to
COMPARISON BETWEEN DIFFERENT SCENARIOS (I= 10 A) (∆y = 0m)

ON POTENTIAL [V]

HORIZONTAL POSITION [m]

Observation line

Well casing

X=0

X=75m

∆y

Anode

BARE
WET
DRY
MIX1
Concluding remarks

- Examples have been presented showing how modelling can provide critical information to improve the design, operation or management of cathodic protection systems over the life of the asset.
- The calculation performance of these types of models is superior with respect to a multi-domain approach since:
  - The number of DOF does not increase with the number of layers.
  - The layer thickness does not constrain the size of the elements in the mesh.
  - The model construction is simplified, since the geometry of the structures immersed in the electrolyte does not need to consider the presence of interface surfaces between layers.
- The modeling approach can be used in conjunction with measurement techniques such as surface E-LogI measurements and CPET logs to improve the reliability of CP designs.
- Sensitivity studies can be made with the simulation model to gain better understanding of the relationship between different CP variables to ensure optimum protection is provided.