AUTOMATIC FATIGUE CRACK GROWTH

S.C. Mellings
BEASY
Southampton, United Kingdom

J.M.W. Baynham
BEASY
Southampton, United Kingdom

ABSTRACT

One of the critical requirements of fatigue crack growth simulation is calculation of the remaining life of a structure under cyclic loading. This paper presents a method which predicts the remaining fatigue life of a part, and gives information on the eventual mode of failure.

The path of a growing crack needs to be understood so that informed assessment can be made of the structural consequences of eventual fast growth, and the likelihood of leakage and determination of leakage rates. For these reasons the use of standard handbook solutions for crack growth is generally not adequate, and it is essential to use the real geometry and loading.

The reasons for performing such simulation work include preventive investigations performed at the design stage, forensic investigations performed after failure, and sometimes forensic investigations performed during failure-when the results provide input to the planning of remedial work.

This paper focuses on the 3D simulation of cracks growing in metal structures exposed to cyclic loading, and explains the techniques which are used. The loading might arise from transients of pressure or other mechanical forces, or might be caused by thermal-stress variations. The simulation starts from an initial crack which can be of any size and orientation. The relevant geometry of the cracked component is modelled, and the loading is identified using one or more load cases together with a load spectrum which shows how the loading cycles. The effects of the crack are determined by calculating stress intensity factors at all positions along the crack front (it would be called the crack tip if the modelling was performed in 2D). The rate and direction of crack growth at each part of the crack front are calculated using one of the available crack growth laws, together with appropriate material properties. The effects of such growth are accumulated over a number of load cycles, and a new crack shape is determined. The process is repeated as required.

The use of multi-axial and mixed mode techniques allows the crack to turn as a result of the applied loading, and the resulting crack path is therefore a consequence of both the detail of the geometry and the loading to which the structure is subjected.

Gas or other fluid pressures acting on the crack faces can have significant impact, as can the contact between opposing crack faces when a load case causes part of the crack to close.

NOMENCLATURE

\[
\begin{align*}
\frac{da}{dN} & \quad \text{Crack growth rate} \\
\Delta K & \quad \text{Effective stress intensity factor range} \\
K_I & \quad \text{Mode 1 stress intensity factor} \\
K_{II} & \quad \text{Mode 2 stress intensity factor} \\
K_{III} & \quad \text{Mode 3 stress intensity factor} \\
K_{eff} & \quad \text{Effective stress intensity factor} \\
f & \quad \text{Crack closure function} \\
R & \quad \text{Load ratio (min/max)} \\
\Delta K_{th} & \quad \text{Threshold stress intensity factor range} \\
K_c & \quad \text{Critical stress intensity factor} \\
C, n, p, q & \quad \text{Nasgro growth material parameters}
\end{align*}
\]

a \quad \text{Crack depth} \\
c \quad \text{Crack half width}
INTRODUCTION

The analysis of cracks in pressure vessels is a critical part of the design process, with the ideal being for the part to exhibit “leak before failure”, rather than catastrophic failure before leak. However to ascertain the mode of failure requires understanding of the way in which initial flaws grow and perhaps coalesce under the applied loading. This understanding is difficult to obtain experimentally in the general case, and is also difficult if not impossible to establish by use of standard reference solution calculations.

This paper presents an alternative to the standard reference solution approach, in which the real geometry of the structure and cracks is used to determine stress intensity factors along each entire crack front, and the resulting mixed mode fracture results are used to simulate growth of the crack. Previous papers by the authors have applied these techniques to other model types, here these have been extended to the study of pipeline and pressure vessel systems.

BOUNDARY ELEMENT ANALYSIS

The numerical analysis presented in this paper is based on the Boundary Element Method (BEM) rather than the ubiquitous Finite Element Method (FEM).

The BEM has significant advantages for mathematical modelling of fracture, including that only the surfaces of the part need to be meshed. The crack, as a surface of the model, is meshed as well, using a layer of elements on both the top and the bottom surfaces of the crack. Figure 7 shows a view from the inside of a structure, with elements on the crack surfaces shown in green, and those on other surfaces shown in blue. The half-penny-shaped crack has a roughly semi-circular crack front.

The simplicity of the mesh required to represent behaviour of cracks is a major advantage when using BEM for fracture analysis. The method allows very refined meshing near the crack front without any difficulty at all. By contrast FE meshes often suffer from the unwanted side-effect that the refinement tends to propagate through the volume of the structure. The simplicity provided by the BEM can especially be appreciated during the growth of a crack, since the only parts of the model that are affected are the surface mesh on the crack and the mesh on the immediately adjacent surfaces. In particular with BEM, it is not necessary to recreate the mesh of the entire structure.

Additional advantages of the BEM are first that it is a mixed method which calculates both displacement components and stress components directly, and secondly that these quantities are calculated at positions on the surface of the structure. Some other methods derive stresses by differentiation of displacement, and often calculate results at “Gauss points” inside the volume, thereafter extrapolating to the surface. It is well known that differentiation tends to dilute accuracy, and in pressure-vessels, the highest stresses are generally at the surfaces, with sometimes very rapid variation near the surface, which means that extrapolation to the surface can be very misleading.

The end result is that BEM provides a methodology which fits very conveniently with fracture and crack growth simulation requirements, and makes it feasible to perform automatic fatigue crack growth modelling.

In this paper a specific form of the BEM is applied, involving use of dual elements on the crack. These allow the greatest simplicity of modelling, since the same element geometry can be used to represent the behaviour on both sides of the crack, hence the name “dual”. From a mathematical viewpoint, a node (at which problem variables are defined) on one side of the crack is at the same position as the node on the corresponding dual element on the other side of the crack. This situation would ordinarily prevent a solution from being obtained, but the dual BEM avoids this difficulty by using an influence equation (for the dual node) which is derived in a different way. The examples shown in this paper were modelled using the BEASY simulation package, which additionally allows selection of an initial crack from a library, and automatic insertion of the crack into a BEM model of an un-cracked structure.

The benefits of mathematical modelling are obtained when the method is applied to situations which are not covered either by standards, previous experience, reference solutions, or experiment.

After a description of the automatic crack growth process, the remainder of this paper first establishes the benchmark that the BEM process discussed does correspond to known reference solutions. The paper then goes on to apply the method to a selection of solutions which are not easily solved any other way.

THE AUTOMATIC CRACK GROWTH PROCESS

In this paper the automatic crack growth process uses a toolkit for fracture analysis in which a crack can be added to an existing, un-cracked, BEM mesh. In the examples presented here the mesh was generated from a geometric model of a pipe. An example of the initial mesh is shown in Figure 1 where the mesh of boundary elements has been created, but the crack has not yet been initiated in the part.
The simulation process continues with selection of the required initial crack shape from a library of shapes. This is achieved within a GUI which leads the user through the various necessary tasks, such as choosing the size of the initial crack and its position and orientation within the component. Figure 2 shows an example of the appearance of the GUI during selection of a crack shape. Other necessary tasks include selection of a “growth distance” used during simulation of fatigue growth, and selection of appropriate fatigue growth data and a crack growth rate equation. (In all the examples in this paper the NASGRO equation and material “E2AA13AB1” have been used during simulation of fatigue crack growth). Optionally other settings can be adjusted to control mesh density, number of J-integral calculations to be performed along each crack front and so on.

Having defined all the necessary data, the simulation process is started, the GUI can be closed, and the steps required for fatigue crack growth then proceed automatically.

The very first step involves placement of the required initial crack into the mesh of the un-cracked component, and modification of the mesh on the crack and nearby surfaces of the component so that it is suitable for the fracture calculation. The mesh after completion of this step is as shown in Figure 3, with detail around the crack shown in Figure 4.

In the next step of the process, the mesh with the initial crack is first used compute stresses and displacements, and then to determine the stress intensity factors. This computation is usually performed using a J-integral method, however the crack opening displacement method can also be used[2].

Next the fatigue behaviour is simulated, to “grow” the crack. This involves use of the stress intensity factors and load variation details to select appropriate direction and distance of growth. This calculation is made at multiple positions along the crack front, and a new position of the crack front, for the grown crack, is thus predicted.

In the next step, the predicted new crack front position is used together with the old crack, to define the surface of the new grown crack.

To complete the cycle, the final step involves placement of the new grown crack into the mesh of the un-cracked component, and as before modification of the mesh on the crack and nearby surfaces of the component so that it is suitable for the fracture calculation.

The process, shown diagrammatically in Figure 5, is continued until one of several stopping criteria is met. These criteria include for example occurrence of critical stress intensity factor, or the attainment of the required total growth distance.
In this initial example, a crack with a semi-circular front has been initiated in a pipe which has diameter 600mm and wall thickness 20mm. The pipe has internal pressure cycling from zero to 40 MPa. The crack is oriented with its surface intersecting the inside surface of the pipe, and the resulting “breakout line” is parallel to the axis of the pipe. The initial crack is planar, with its surface at right angles to the plane which is, at the breakout line, tangential to the inner surface of the pipe. The initial crack has radius 0.5mm. The un-cracked pipe model is shown in Figure 6.

Figure 6 Pipeline model

Figure 7 shows the crack after fatigue growth has been performed such that the “a” dimension (shown in annotation of Figure 9) has increased to ~2 mm. More details on how this crack growth process is performed is given later in the paper. As the crack grows, it remains planar, but becomes slightly non-circular, as can be seen more clearly in Figure 8 which shows both the crack at “a” dimension ~13 mm and the position of the outer surface of the pipe, towards which the crack is growing.
The simplicity of this first example is such that it is possible to compare fatigue results from the BEM simulation with results computed using NASGRO 5\cite{8}. The NASGRO fatigue case is shown in Figure 9. This picture is taken from the NASGRO 5 NASFLA program and show the analysis case used. In this analysis the through thickness stress values are set to be zero (i.e. $S_i(x)=0$, $i=1,2,3$) and only internal pressure is used in the analysis.

In the NASGRO crack growth law, the crack growth rate is given by the equation:

$$
\frac{da}{dN} = C \left( \frac{1-f}{1-R} \right)^n \left( 1 - \frac{\Delta K_{th}}{\Delta K} \right)^p \left( 1 - \frac{\Delta K}{(1-R)K_i} \right)^q
$$

Equation 1 NASGRO crack growth law

In this equation most of terms used (C, n, p, q, etc) are material parameters that are defined to match to test data. Some of these parameters are described in the nomenclature at the start of this paper. The value $\Delta K$ is the stress intensity factor range and the value R is the ratio of the minimum SIF to the maximum SIF. This equation describe the law for a generic mode 1 model, however for a mixed model analysis, a combination of the modal results is used. This is described later in this paper. In the analysis tool used, the NASGRO is included in the analysis code and the crack is grown using the same material parameters as defined in the NASGRO 5 analysis.

In all the examples presented in this paper, the simulation has been carried out using fatigue parameters for a pressure vessel/piping steel property defined in the NASGRO 5 database with material label E2AA13AB1. This material property data defines the parameters used in the NASGRO crack growth rate equation shown above. For more information on this growth law refer to the NASGRO 5 documentation\cite{8}.

The fatigue life results from the NASGRO analysis (using NASGRO 5) have been compared in Figure 10 with those obtained using the BEM mathematical modelling. The figure shows variation of crack size (both the “a” and the “c” dimensions) with the number of cycles of loading. The shapes of the graphs of crack length versus number of cycles in both cases show that the crack is growing faster along the length of the pipe than through the thickness of the pipe.

The stress intensity factors determined by NASGRO and in the BEM simulation, shown in Figure 11 for four different positions of the crack front, confirm the reason for the growth being faster at the “c” position than at the “a” position, which is simply that the SIF is higher at the “c” position than at the “a” position.

The apparently much higher SIF at the “c” position calculated by NASGRO at $a=6$ mm and $a=12.5$ mm is a consequence of the accumulated difference in the crack shape.

The results at $a=19.04$ mm suggest that while the BEM is representing the effect of the now very small remaining wall thickness, the NASGRO result is not.
In this second example it so happens that the crack grows with a shape similar to the first, as shown in Figure 12. However the effect of the pressure on the crack surfaces can be very clearly seen in Figure 13, which shows that the crack grows at an accelerated rate. With pressure acting on the crack surfaces, the crack reached “a” dimension=13mm after 62184 loading cycles, whereas without pressure on the crack surface the same size was reached after 86862 cycles. This ~30% reduction of life is significant, and highlights the importance of using a truly representative methodology.

In this third example two co-planar semi-circular cracks of radius 1.0 mm, separated by ~27.4 mm are initiated, in a pipe model for which all other details are as in the first example.

By St Venant’s principle, it might be estimated that the cracks will start to influence each other at separation equal to about 3 times the crack “diameter”, but this estimate might be affected by the ratio of the crack diameter to the pipe wall thickness.

Initially the cracks grow do not interfere, as suggested by contours of maximum principal stress shown in Figure 14. However at the stage when the cracks have “c” dimension 9mm, it is very clear that the stress fields are interfering, as can be seen from the contours shown in Figure 15.
The interference between the cracks also becomes apparent from the stress intensity factors, and the cracks begin to grow towards each other more quickly, as can be seen in Figure 16.

Eventually the two cracks coalesce, but this cannot be handled automatically and some user intervention is required to join the cracks. This is achieved by creating a surface that combines the two cracks, as shown in Figure 17, and using this to define a new “initial” crack.

The coalesced crack is then used to restart the automatic growth process. Figure 18 shows the crack after placement into the mesh of the un-cracked component, and Figure 20 shows the crack when it has grown further. The “re-entrant corner” in the crack front has the effect of locally increasing the stress intensity factors which in turn causes the crack to grow quickly there. This effect can be seen in Figure 19 which shows the biggest growth distances in the region of the re-entrant corner. The result is that the re-entrant corner tends to disappear, as shown in Figure 20.
MIXED MODE CRACK GROWTH

In all the examples shown so far, the initial cracks have been oriented so they are normal to the direction of maximum principal stress. In such a situation it is expected that the cracks grow in-plane, and so should remain flat, and this is demonstrated in the BEM results.

However in real situations it will generally be the case that either the stress field will vary throughout the part or an initial defect will not be orientated normal to the maximum principal stress direction. When this occurs the calculated stress intensity factors will in general show non-zero values for the $K_I$ (mode 1), $K_{II}$ (mode 2) and $K_{III}$ (mode 3) results. These so-called mixed mode results will cause the crack to turn as it grows.

Within BEASY the independent modal results are computed using a decomposition of the J-Integral as described by Rigby\textsuperscript{[9,10]}.

The next example, which exhibits mixed-mode grow, is based on the first example used elsewhere in this paper, but with an initial crack that is rotated through 60˚ relative to the principal stress direction, as shown in Figure 21.

The stress intensity factors for the initial crack under maximum load are shown in Figure 22. The figure shows that the Mode 2 ($K_{II}$) SIF value changes from a negative value at one end of the crack to a positive value at the other end. This indicates the crack will turn in opposite directions at either end of the crack.

As before, the process involves prediction of growth distances and directions, for some number of cycles of loading. The method used in this case to determine the growth direction is based on calculation of the angle to the crack surface at which a growing crack will minimise the strain energy density. This direction is computed at multiple positions along the crack front. The way in which the calculated angle varies along the crack front is shown in Figure 23.

The crack growth rate for a mixed mode condition is computed by evaluating an effective stress intensity factor. Various methods of combining the three stress intensity factors are available and in this example the method chosen is shown in the equation:

\[
K = \sqrt{(K_I + |K_{III}|)^2 + 2K_{II}^2}
\]

**Equation 2 Computation of effective SIF**

This equation is a derivative of the Tanaka function and was used by Mi & Aliabadi\textsuperscript{[4]}.
The surface of the new grown crack follows all the angle changes as the crack grows. After the first step of simulated growth, the crack appears as shown in Figure 24. The simulation process continues as before, resulting in stress intensity factors for the newly grown crack as shown in Figure 25.

Figure 24 The crack shape after first growth step

Figure 25 SIF results after growth step 1

Figure 25 shows that the magnitudes of the mode 2 and 3 SIFs have now dropped significantly compared with the values in the previous step, and also shows that the mode 1 SIF has risen slightly. This effect is typical of a crack growing under uniaxial loading. The consequence is that the crack will now grow in a predominantly mode 1 direction, and this is confirmed by the shape of the crack after the second growth step, shown in Figure 26, where it can be seen there was a much smaller angle change. The crack shape after several growth steps is shown in Figure 27.

Figure 26 The crack shape after 2nd growth step

Figure 27 The crack shape after 6th growth step

Figure 27 indicates that the crack initially grew in a zig-zag fashion. This is an inevitable consequence of using a finite growth distance, and means that sometimes the path of the growing crack in the simulation has “overshot” the real crack growth path. (The problem would go away if growth corresponding to a single load cycle could be used, but in practise this would be very computationally expensive).

The effect of the overshoot is that the next growth direction will turn the crack in the opposite direction. The overshoot and self-correction can be clearly seen in Figure 27, with the overshoot gradually disappearing as the crack grows.

The presence of the zig-zag pattern does however indicate that the crack is growing too far during the initial stages of growth. It is possible to control the growth distances, and Figure 28 shows the crack growth path which results when the growth distances have been reduced, in this case from the previous 1.5mm to 0.5mm per step.

It can be seen that this has removed most of the oscillation from the crack path and it is clear the crack has initially turned but then grown in a mode 1 direction. If required the crack growth distances could be increased once the crack growth direction has stabilised, but of course the smaller growth distances provide more accurate information about the change in the stress intensity factors.

Figure 28 Crack grown 2.5mm using smaller increments

CONTACT ON CRACK FACES

Finally we will consider another loading applied to the same pipe model. Now, rather than the applied pressure loading, a torque will be simulated along the pipe. In this analysis the loading will be applied in alternating directions, using two load cases.

This situation can be simulated using the normal process, but special techniques must be activated to take account of the
contact between opposing faces of the crack which can occur under this loading. If the contact is not simulated, one of the load cases results in negative mode 1 stress intensity factors along the crack front as shown in Figure 29.

**Figure 29 SIF values from “compressive torque load case”**

The other load case results in positive mode 1 SIF, as shown in Figure 30.

**Figure 30 SIF values from “tensile torque load case”**

The negative mode 1 SIF values imply that the crack faces are interfering, which can be allowed mathematically but in reality would not occur. Rather the crack faces would be in contact and load would be transferred across the crack. This effect can be included in the simulation simply by turning on use of contact algorithms on the crack faces.

Figure 31 shows the effect of applying contact to the crack faces, which is that the mode 1 SIF value is now nearly zero, and crack growth will now be dominated by the mode 2 SIF.

**Figure 31 SIF from compressive load with contact**

Although contact algorithms were turned on, the 2nd load case tends to open the crack, so there is no contact. Figure 32 shows that the resultant SIF values are identical to those in the model which did not have contact algorithms turned on.

**Figure 32 SIF from “tensile torque load case” with contact**

These SIF values can now be used to grow the crack, but alternative algorithms are now required, because the loading has now become “multi-axial” – ie the load cycle cannot be achieved by scaling a single stress field.

Without contact on the crack faces the crack grows as shown in Figure 33 whereas with contact algorithms turned on it grows as shown in Figure 34.

**Figure 33 Crack growth (step 1) without crack face contact**
This is quantified further in the graphs shown below showing the difference in the crack growth angle (Figure 35) and crack growth rate (Figure 36) for the two analysis methods.

It has been shown that the methodology takes into account the real stress fields which occur in practise, including effects of section thickness change, use of the real component geometry, use of the real crack geometry, pressure loading on crack surfaces, interference between cracks, mixed-mode loading, contact on crack surfaces, and multi-axial loading.

It has been shown that the methodology can show how a crack will grow without being constrained by standard reference solutions.

By extension, it can be concluded that this technique allows investigations to be performed into the grown crack shapes and enables effects of design changes to be studied in a realistic manner.

Although not discussed, the technique of applying loading to the crack faces can be extended to allow effects of residual stresses to be studied. This is necessary for study of effects on crack growth of different surface treatments or different methods of welding.

CONCLUSION

A methodology has been presented in this paper which can be applied to crack growth in real components.

REFERENCES


[8] NASGRO5 Userguide, Southwest Research Institute, 2008
